

Descriptive theory of deterministic chaos

O. M. Sharkovsky

Institute of Mathematics, NAS of Ukraine

Descriptive theory of sets is a classical section of mathematics, which arose at the beginning of the last century. The talk proposes the basis of the **descriptive theory of chaos**.

We consider dynamical systems on a compact X , generated by a continuous map $f : X \rightarrow X$, mainly in the case of when X is an interval $I \subset R$.

The asymptotic behavior of every trajectory is usually determined through the so-called ω -limit set, or, more simply, **the attractor** of this trajectory. The set of all trajectories attracted by the same attractor is called **the basin of this attractor**.

Dynamical system if its topological entropy is positive :

- 1) has a lot of different attractors of trajectories, namely, the continuum of attractors;
- 2) basins of most attractors have a very complex structure, namely, they are sets of the 3rd class in the terminology of the descriptive theory of sets;
- 3) basins of different attractors are very intertwined and they can not be separated from each other by open or closed sets, but only by sets of the 2nd class of complexity, and
- 4) in the space of all closed subsets of the state space (with the Hausdorff metric), the set of all attractors is an attractor net (network) whose cells are formed by Cantor sets whose points are themselves attractors.

Most of the results presented in the talk were obtained and translated into English back in the 60's of the last century.

[1] A.N.Sharkovsky, *On attracting and attracted sets*, Soviet Math. Dokl. **6**, 1965, 268-270 (transl. from Dokl. Akad. Nauk SSSR **160**, 1965, 1036-1038).

[2] A.N.Sharkovsky, *A classification of fixed points*, Amer. Math. Soc. Transl. (2) **97**, 1970, 159-179 (transl. from Ukrain. Mat. Zh. **17**(5), 1965, 80-95).

[3] A.N.Sharkovsky, *Behavior of a mapping in the neighborhood of an attracting set*, Amer. Math. Soc. Transl. (2) **97**, 1970, 227-258 (transl. from Ukrain. Mat. Zh. **18**(2), 1966, 60-83).

[4] A.N.Sharkovsky, *Partially ordered system of attracting sets*, Soviet Math. Dokl. **7**, 1966, 1384-1386 (transl. from Dokl. Akad. Nauk SSSR **170**, 1966, 1276-1278).

[5] A.N.Sharkovsky, *Structure of an endomorphisms on ω -limit sets*, Intern. Math. Congress (Moscow, 1966), Sect.6, Abstracts, 51.

[6] A.N.Sharkovsky, *Attractors of trajectories and their basins*, Naukova Dumka, Kiev, 2013, 320p. (in Russian).