Syllabi of M.Sc. (Mathematics/Applied Mathematics)

January 9, 2017

Contents

Note: All Courses are of FOUR credits

I Semester (Math./Appl.Math.)

1 Real Analysis-I (Code: MM401)

Real number system and its structure, infimum, supremum, Dedekind cuts. (Proofs omitted)

Sequences and series of real numbers, subsequences, monotone sequences, limit inferior, limit superior, convergence of sequences and series, Cauchy criterion, root and ratio tests for the convergence of series, power series, product of series, absolute and conditional convergence, metric spaces, limits in metric spaces.

Functions of a single real variable, limits of functions, continuity of functions, uniform continuity, continuity and compactness, continuity and connectedness, types of discontinuities, monotonic functions, infinite limit and limit at infinity.

Differentiation, properties of derivatives, chain rule, Rolle's theorem, mean value theorems, l'Hospital's rule, derivatives of higher order, Taylor's theorem.

Sequences and series of functions, pointwise and uniform convergence, continuity of the uniform limit of continuous functions, uniform convergence and differentiability, Dini's theorem, equicontinuity, pointwise and uniform boundedness, Arzela-Ascoli's theorem, Weierstrass approximation theorem, space filling curve, continuous but nowhere differentiable function.

Fourier series: Dirichlet kernel, pointwise convergence of Fourier series.

References

[1] Goldberg, Richard R., *Methods of Real Analysis*, second edition, John Wiley & Sons, Inc., New York-London-Sydney, 1976.

[2] Rudin, Walter, *Principles of Mathematical Analysis*, third edition, International Series in Pure and Applied Mathematics. McGraw-Hill Book Co., New York-Auckland-Düsseldorf, 1976.

[3] Bartle, Robert G., *The Elements of Real Analysis*, second edition, John Wiley & Sons, New York-London-Sydney, 1976. (for Fourier Series)

[4] Ross, Kenneth A., *Elementary Analysis. The Theory of Calculus*, second edition, in collaboration with Jorge M. López, Undergraduate Texts in Mathematics, Springer, New York, 2013.

2 Linear Algebra (Code: MM402)

Matrices: Elementary operations, reduced row-echelon form, consistency of a system of equations, solutions of systems of equations, homogeneous system, inverse of a matrix, determinants, Cramer's rule.

Vector spaces and subspaces, linear independence of vectors, basis.

Linear transformations and matrices, kernel, nullity theorem, rank of a matrix, similarity, characteristic polynomials, eigenvalues, theorems on eigenvalues and eigenvectors, Cayley-Hamilton theorem, properties of characteristic polynomials, direct sum, Jordan form and diagonalization.

Inner Product spaces , C-S inequality, triangle inequality, orthonormal basis, Gram-Schmidt construction of orthonormal basis.

Bilinear and quadratic forms, properties.

References

[1] Hoffman, Kenneth and Kunze, Ray, *Linear Algebra*, second edition, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1971.

[2] Rao, A. R. and Bhimashankaram, P., *Linear Algebra*, second edition, TRIM series, 2000.

3 Algebra I (Code: MM403)

Boolean Algebras: Sets, relations, functions, equivalence relations, equivalence classes, partial order relations, posets, chains, anti-chains, Zorns lemma, lub and glb in a poset, Lattice, distributive lattice, universal upper/lower bonds, complemented distributive Lattices, Boolean lattices/algebra, Finite Boolean latices.

Groups: Groups and Rings: Introduction, Semi-groups, Groups, Subgroup, Generators and Evaluation of Powers, Cosets and Lagrange's Theorem, Permutation Groups, Normal Subgroups, Quotient groups, Group homomoprhisms, Automorphisms, Isomorphisms, Fundamental theorems of group homomoprhisms, Cayley's Theorem, Group actions, Burnsides Theorem, Sylow's Theorems 1st, 2nd, 3rd and their applications.

Structure theorem for finite Abelian groups, Composition series Jordan-Hölder theorem, Nilpotent and solvable groups.

References

[1] Michael Artin, *Algebra*, 2nd ed. Pearson, Upper Saddle River, NJ, 2011

[2] I.N. Herstein, *Topics in Algebra*, 3rd ed. Wiley, New York, 1996.

[3] Dummit and Foote, *Abstract Algebra*, 3rd ed. Wiley, New York, 2003.

[4] Halmos, *Nave Set Theory*, Springer, New York, 1991.

4 Elements of Probability and Statistics (Code: MM404)

Random experiments, sample spaces, sets, events, algebras. elements of combinatorial analysis; classical definition and calculation of probability, independence of events.

Random variables, distribution functions, moments, probability and moment generating functions, independence of random variables, inequalities.

Introduction to various discrete and continuous random variables, limiting distributions of some random variables, distributions of functions of random variables.

Bi-variate distributions, conditional and marginal distributions, conditional expectation and variance, co-variance and correlation co-efficient, bivariate moment generating functions.

Elementary understanding of data: Frequency curves, empirical measures of location, spread, empirical moments, analysis of bivariate data; fitting of distributions.

Sampling distributions, Chi-square, t, F.

References

[1] Feller, W., *Introduction to Probability Theory and its Applications*, third edition , Wiley Eastern, 1978.

[2] Ross, S., *A First Course in Probability*, sixth edition , Pearson Education, 2007.

[3] Prakasa Rao, B. L. S., *A First Course in Probability and Statistics*, World Scientific, 2009.

5 Numerical Methods and Programming (Code: MM405)

Students need to write programmes for the algorithms that they learn in Module-2 from time to time. Topics in Module-1 and Module-3 should support topics in Module-2.

MODULE-1 (At least 10 hands on sessions):

Flow charts and algorithms, sample C-Programmes: compilation and execution of the programmes.

C-alphabet: ASCII Character set, basic data types, variables and constants in C.

Operators in C: Hierarchy and associativity.

Flow control instructions in C: Decision control (if- else), loop control (for, while, do-while), case control (switch).

The break and continue statements, functions, arrays, structures (user defined data types).

MODULE-2 (NUMERICAL COMPUTATION):

Representation of integers and fractions, fixed point and floating point arithmetics, error propagation, loss of significance, condition and instability, computational method of error propagation.

Root finding: bisection method, secant method, regula-falsi method, Newton-Raphson method, LU decomposition, Gauss elimination with and without pivoting, Gauss-Jacobi method, Gauss-Seidel method, Power method, Jacobi method to find eigenvalues.

Interpolation: Lagrange's interpolation, Newton's divided difference interpolation (forward, backward), Newton-Gregory formulae, Sterling's formula.

Numerical integration: Newton-Cotes (closed type formulae)-trapezoidal rule, Simpson's $\frac{1}{3}$-rd rule, Simpson's $\frac{3}{8}$-th rule.

MODULE-3 (At least 10 hands on sessions):

MATLAB/Octave, implementation of algorithms which are in Module-2.

References

[1] Yashavant P. Kanetkar, *Let Us C*, BPB publications, 13$^{\text{th}}$ edition, 2012.

[2] Yashavant P. Kanetkar, *Test Your C Skills*, BPB publications, fifth edition, 2009.

[3] Conte, S. D. and deBoor, C., *Elementary Numerical Analysis - An Algorithmic Approach*, third edition, McGraw Hill, 1981.

[4] Henrici, P., *Elements of Numerical Analysis*, John Wiley & Sons, 1964.

[5] Froberg, C. E., *Numerical Mathematics - Theory and Computer Applications*, The Benjamin Cummings Pub. Co. 1985.

[6] Quarteroni, A.; Saleri, F. and Gervasio, P., *Scientific Computing with MATLAB and Octave*, third edition, Springer, 2010.

[7] Rudra Pratap, *Getting Started with MATLAB 7: A Quick Introduction for Scientists and Engineers*, Oxford University Press, 2005.

[8] Stoer, J. and Bulirsch, R., *Introduction to Numerical Analysis*, Texts in Applied Mathematics, Springer, 2002.

II Semester (Math./Appl.Math.)

6 Real Analysis II (Code: MM451)

Functions of several variables, directional derivative, partial derivative, total derivative, Jacobian, chain rule and mean value theorems, higher derivatives, interchange of the order of differentiation, Taylor's theorem, inverse mapping theorem, implicit function theorem, extremum problems, extremum problems with constraints, Lagrange's multiplier method.

Multiple integrals, properties of integrals, existence of integrals, iterated integrals, change of variables.

Curl, gradient, div, Laplacian in cylindrical and spherical coordinates, line integrals, surface integrals, theorems of Green, Gauss and Stokes.

References

[1] Apostol, Tom M., *Mathematical Analysis*, second edition, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1974. (Chapter 6,7,10 and 11.2).

[2] Apostol, Tom M., *Calculus. Vol. II: Multi-variable Calculus and Linear Algebra, with Applications to Differential Equations and Probability*, second edition, Blaisdell Publishing Co. Ginn and Co., Waltham, Mass.-Toronto, Ont.-London, 1969.

[3] Munkres, James R., *Analysis on Manifolds*, Addison-Wesley Publishing Company, Advanced Book Program, Redwood City, CA, 1991.

[4] Spiegel, Murray R., *Schaum's Outline of Vector Analysis*, Schaum's Outline Series, 1959.

[5] Spivak, Michael, *Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus*, W. A. Benjamin, Inc., New York-Amsterdam, 1965.

[6] Moskowitz, M., and Paliogiannis F., *Functions of Several Real Variables*, World Scientific, 2011.

7 Ordinary Differential Equations - I (Code: MM455)

FIRST ORDER DIFFERENTIAL EQUATIONS:

Ordinary Differential Equations, mathematical models, first order equations, existence, uniqueness theorems, continuous dependence on initial conditions, Gronwall's inequality and applications.

SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS:

Wronskian, explicit methods to find solutions, method of variation of parameters; power series solutions: ordinary points, regular singular points, irregular singular points and Frobenius methods; special functions: Legendre and Bessel functions, properties.

Oscillation theory, qualitative properties of solutions, Sturm separation and comparison theorems.

Two-point boundary value problems: Sturm-Liouville equations, Green's functions, construction of Green's functions, nonhomogeneous boundary conditions, eigenvalues and eigenfunctions of Sturm-Liouville equations, eigenfunction expansions.

SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS:

Existence and uniqueness theorems; homogeneous linear systems, fundamental matrix, exponential of a matrix, nonhomogeneous linear systems, linear systems with constant coefficients,

NONLINEAR DIFFERENTIAL EQUATIONS :

Volterra Prey-Predator model.
Phase plane analysis : Autonomous systems, types of critical points, stability for linear systems with constant coefficients, stability of nonlinear systems, method of Lyapunov for nonlinear systems, simple critical points, Poincaré's theorem, limit cycles, statement of Poincaré-Bendixson theorem, examples.

References

[1] Simmons, George F., *Differential Equations with Applications and Historical Notes*, International Series in Pure and Applied Mathematics, McGraw-Hill Book Co., New York-Düsseldorf-Johannesburg, 1972.

[2] Birkhoff, Garrett and Rota, Gian-Carlo, *Ordinary Differential Equations*, fourth edition, John Wiley & Sons, Inc., New York, 1989.

[3] Coddington, Earl A. and Levinson, Norman, *Theory of Ordinary Differential Equations*, McGraw-Hill Book Company, Inc., New York-Toronto-London, 1955.

[4] Perko, Lawrence, *Differential Equations and Dynamical Systems*, Springer, New York, third edition, 2001.

[5] Ross, Shepley L., *Introduction to Ordinary Differential Equations*, fourth edition, John Wiley & Sons, Inc., New York, 1989.

[6] Cronin, Jane, *Ordinary Differential Equations. Introduction and Qualitative Theory*, third edition, Pure and Applied Mathematics, 292. Chapman & Hall/CRC, Boca Raton, FL, 2008.

[7] Hirsch, Morris W., Smale, Stephen and Devaney, R. L., *Differential Equations, Dynamical Systems and an Introduction to Chaos*, Academic Press, 2004.

8 Measure and Integration (Code: MM452)

Riemann-Stieltjes integral, Riemann's condition, linear properties of integration, necessary conditions for existence of Riemann-Stieltjes integrals, sufficient conditions for existence of Riemann-Stieltjes integrals, reduction to Riemann integral, change of variable in a Riemann-Stieltjes integral, comparison theorems, mean value theorems for Riemann-Stieltjes integrals, integral as a function of the interval, fundamental theorem of integral calculus, improper integrals and tests for their convergence, absolute convergence.

σ-algebras of sets, Borel subsets of $\mathbb{R}$, Lebesgue outer measure and its properties, σ-algebra of measurable sets in $\mathbb{R}$, non-measurable set, example of measurable set which is not a Borel Set, Lebesgue measure and its properties, Lebesgue-Stieltjes measure, measurable function, pointwise convergence and convergence in measure, Egoroff theorem, Lebesgue integral, Lebesgue criterion of Riemann integrability, monotone convergence theorem, Lebesgue dominated convergence theorem, Fatou's Lemma, convergence theorems, differentiation of an integral, absolute continuity with respect to Lebesgue measure, Lebesgue integral in the plane, introduction to Fubini's theorem, L_p-spaces.

References

[1] Apostol, Tom M., *Mathematical Analysis*, second edition, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1974.

[2] Bartle, Robert G., *The Elements of Real Analysis*, second edition, John Wiley & Sons, New York-London-Sydney, 1976.

[3] Royden, H. L., *Real Analysis*, third edition, Macmillan Publishing Company, New York, 1988.

[4] de Barra, G., *Measure Theory and Integration*, New Age International Pvt. Limited, 1981.

9 Topology - I (Code: MM453)

Definition of topologies in terms of open sets, neighbourhood system, closed sets and closure operations and their equivalence, points of accumulation, interior, exterior and boundary points.

Base and sub-base of a topology, subspace, product space, quotient space, continuous, open and closed maps, homeomorphism, convergence of sequences, separation axioms, separability, Lindeloff space, Urysohn's metrization theorem, compactness, local compactness, sequential and countable compactness, characterization of compact metric spaces, Tychonov theorem, one point compactification, connectedness and local connectedness and path connectedness.

References

[1] Dugundji, James, *Topology*, Allyn and Bacon Series in Advanced Mathematics, Allyn and Bacon, Inc., Boston, Mass.-London-Sydney, 1978.

[2] Munkres, James R, *Topology: A First Course*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1975

[3] Kelley, John L., *General Topology*, Graduate Texts in Mathematics, No. 27, Springer-Verlag, New York-Berlin, 1975.

10 Algebra-II (Code: MM454)

Basic concepts in rings, ideals, homomorphism of rings, quotients with several examples. Euclidean domains, principal ideal rings/domains, factorization domains and unique factorization domains.

Eisenstein's irreducibility criterion and Gauss's lemma. Modules, Modules over PIDs, Modules with chain conditions, Hilbert Basis Theorem.

Field extensions, algebraically closed fields, finite fields, Ruler and compass techniques etc.

References

[1] Artin, Michael, *Algebra*, Prentice Hall, Inc., Englewood Cliffs, NJ, 1991.

[2] Herstein, I.N. *Topics in Algebra*, 3rd ed. Wiley, New York, 1996.

[3] Dummit, David S. and Foote Richard M., *Abstract Algebra*, third edition, John Wiley & Sons, Inc., Hoboken, NJ, 2004.

[4] Jacobson, Nathan, *Basic Algebra, Volume 1*, second edition, W. H. Freeman and Company, New York, 1985.

[5] Musili, C., *Introduction to Rings and Modules*, Narosa, 1992.

III Semester (Math./Appl.Math.)

11 Complex Analysis - I (Code: MM504/AM504)

Algebra of complex numbers, operations of absolute value and conjugate, standard inequalities for absolute value, extended complex plane, spherical representation, and neighborhoods of i and $\mathbb{C}$ as a metric space and its topological properties.

Concept of analytic function via power series and differentiability methods.

The exponential and logarithmic functions, trigonometric functions of a complex variable.

Analytic functions as mappings from $\mathbb{C}$ to $\mathbb{C}$.

Linear fractional transformations and their properties; conformality of a map and elementary conformal mappings, examples.

Complex Integration: Line integrals, rectifiable curves, Cauchy's fundamental theorem for rectangle, disk; index of a closed curve, Cauchy's integral formula, higher derivatives of analytic functions, Cauchy's inequality, Liouville's theorem.

Singularities: Taylor's theorem, removable singularities, zeros and poles, local mapping essential singularities, examples, Weierstrass theorem, the maximum modulus theorem, Schwartz's lemma, Cauchy's residue theorem, evaluation of definite integrals using Cauchy's residue theorem, Argument principles, Taylor Series and Laurent series, expansions, examples.

References

[1] Ahlfors, Lars V., *Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable*, third edition. International Series in Pure and Applied Mathematics, McGraw-Hill Book Co., New York, 1978.

[2] Churchill, Ruel V. and Brown, James Ward, *Complex Variables and Applications*, fourth edition, McGraw-Hill Book Co., New York, 1984.

[3] Conway, John B., *Functions of One Complex Variable*, II, Graduate Texts in Mathematics, 159, Springer-Verlag, New York, 1995.

[4] Narasimhan, Raghavan and Nievergelt, Yves, *Complex Analysis in One Variable*, second edition, Birkhäuser Boston, Inc., MA, 2001.

12 Functional Analysis (Code: MM501/AM501)

Fundamental of metric spaces, completion of metric spaces, normed spaces, Hölder's inequality, Minkowski's inequality, l^p-spaces, Banach spaces, equivalence of norms, Riesz lemma, bounded linear maps, operator norm, separability, Hahn-Banach theorem, dual of normed spaces ($\mathbb{R}^n$, L^p, $C[a, b]$), reflexivity, open mapping and closed graph theorems, uniform boundedness principle.

Inner product spaces, Hilbert spaces, examples, projection theorem, Bessel's inequality, existence of Schauder basis for separable Hilbert spaces, Riesz representation theorems, compact operators, finite rank operators, properties of self adjoint compact operators, spectral theorem.

References

[1] Kreyszig, Erwin, *Introductory Functional Analysis with Applications*, Wiley Classics Library, John Wiley & Sons, Inc., New York, 1989.

[2] Limaye, Balmohan V., *Functional Analysis*, second edition, New Age International Publishers Limited, New Delhi, 1996.

[3] Kesavan, S., *Functional Analysis*, Trim series, Hindustan Book Agency, 2009.

13 Partial Differential Equations - I (Code: MM502/AM502)

FIRST ORDER P.D.E.:

Surfaces and curves, classification of first order P.D.E., classification of solutions, Pfaffian differential equations, quasi-linear equations, Lagrange's method, compatible systems, Charpit's method, Jacobi's method, integral surfaces passing through a given curve, method of characteristics for quasi-linear and nonlinear P.D.E., Monge cone, characteristic strip.

SECOND ORDER P.D.E.:

Origin of second order P.D.E., classification of second order semi-linear p.d.e., Hadamard's definition of well-posedness.

Wave equation: D'Alembert's solution, vibrations of a finite string, existence and uniqueness of solution, Riemann method.

Laplace's equation: Boundary value problems, maximum and minimum principles, uniqueness and continuity theorems, Dirichlet problem for a circle, Dirichlet problem for a circular annulus, Neumann problem for a circle, theory of Green's function for Laplace's equation.

Heat equation: Heat conduction problem for an infinite rod, heat conduction in a finite rod, existence and uniqueness of the solution; Duhamel's principle for wave and heat equations.

Variable separable methods for second order linear partial differential equations.

Classification of semi-linear partial differential equations in higher dimensions; Kelvin's inversion theorem; equipotential surfaces.

References

[1] Fritz, John, *Partial Differential Equations*, second edition, Applied Mathematical Sciences, Vol. 1, Springer-Verlag, Chapter - I, 1978.

[2] Weinberger, H. F., *A First Course in Partial Differential Equations with Complex Variables and Transform Methods*, Wiley, 1965.

[3] Sneddon, Ian, *Elements of Partial Differential Equations*, McGrawHill, NY, 1957; Dover, 2006.

[4] Qing, Han, *A Basic Course in Partial Differential Equations*, ATM Volume 120 , Indian edition, 2013.

[5] McOwen, Robert C., *Partial Differential Equations - Methods and Applications*, second edition, Pearson India, 2006.

[6] Evans, Lawrence C., *Partial Differential Equations*, AMS-GTM, Vol.19, Indian edition, 2010.

[7] Amaranath, T., *An Elemnetary Course in Partial Differential Equations*, second edition, Narosa Publishing House, 2012.

[8] Zauderer, Erich, *Partial Differential Equations of Applied Mathematics*, third edition, Wiley, 2011.

14 Algebra-III (Code: MM505)

Galois theory, Separable, Galois extensions, Fundamental theorem of Galois theory, separable, normal purely inseparable extensions, The Frobenius function of a field of positive characteristic, perfect fields, Solvability of a polynomial by radicals, ruler and compass constructions, transcendental extensions, Luroth's theorem.

Simple Modules, Jordon-Hölder theorem for Modules, Jacobian and Nil radicals, Nakayama Lemma, Chinese remainder theorem, tensor products. Artinian ring implies Noetherian.

References

[1] Michael Artin, *Algebra*, 2nd ed. Pearson, Upper Saddle River, NJ, 2011

[2] Herstein,I.N. *Topics in Algebra*, 3rd ed. Wiley, New York, 1996.

[3] Dummit and Foote, *Abstract Algebra*, 3rd ed. Wiley, New York, 2003.

[4] Lang, Serge, *Algebra*, revised third edition, Graduate Texts in Mathematics, 211, Springer-Verlag, New York, 2002.

[5] Jacobson, Nathan, *Basic Algebra*, Volume 1, second edition, W. H. Freeman and Company, New York, 1985.

[6] Jacobson, Nathan, *Basic Algebra*, Volume 2, second edition. W. H. Freeman and Company, New York, 1989.

[7] Jacobson, Nathan, *Lectures in Abstract Algebra, III, Theory of fields and Galois theory*, Second corrected printing, Graduate Texts in Mathematics, No. 32. Springer-Verlag, New York-Heidelberg, 1975.

15 Numerical Analysis (Code: AM505)

Solution of nonlinear equations: Multi-point iterative methods, fixed point iteration, convergence of methods, polynomial equations, Muller's method, acceleration of convergence.

Solution of linear systems: Error and residual of an approximate solution. condition number, theorems of Gershgorin and Brauer, Jacobi method, Power method for Hermitian matrices, inverse power method, convergence of the methods.

Polynomial interpolation: Existence and uniqueness of an interpolating polynomial, Hermite interpolation, error of the interpolating polynomials, piecewise-polynomial approximation (up to cubic splines).

Numerical integration: Newton-Cotes closed and open type formulae, error, composite rules, adaptive quadrature, extrapolation to the limit, Romberg Integration, properties of orthogonal polynomials, Gaussian quadrature.

Numerical differentiation. .

Solution of O.D.E.: Difference equations, Taylor series method, explicit and implicit methods, single and multi-step methods - forward, backward Euler methods, mid-point formula, modified Euler's method and their convergence, Runge-Kutta methods (up to 2nd order O.D.E.), Predicator-Corrector methods, stability of numerical methods, round-off error propagation and control, shooting methods and finite difference methods for B.V.P. (second and fourth order).

Solution of linear P.D.E. (at most second order) : Derivation of difference equations for transport equation, heat equation, wave equation, Laplace equation, Poisson equation, consistency, initial value problems, P.D.E. with Dirichlet and Neumann boundary conditions, stability, Lax theorem, von Neumann (L^2) stability.

References

[1] Conte, S. D. and deBoor, C., *Elementary Numerical Analysis - An Algorithmic Approach*, third edition, McGraw Hill, 1981.

[2] Henrici, P., *Elements of Numerical Analysis*, John Wiley & Sons, 1964.

[3] Froeberg, C. E., *Numerical Mathematics - Theory and Computer Applications*, The Benjamin Cummings Pub. Co., 1985.

[4] Stoer, J. and Bulirsch, R., *Introduction to Numerical Analysis*, Texts in Applied Mathematics, Springer, 2002.

[5] Press, William H.; Flannery, Brian P.; Teukolsky, Saul A. and Vetterling, William, T., *Numerical Recipes in C: The Art of Scientific Computing*, second edition, 1992.

[6] Quarteroni, A.; Saleri, F. and Gervasio, P., *Scientific Computing with MATLAB and Octave*, third edition, Springer, 2010.

[7] Thomas, J. W., *Numerical Partial differential Equations: Finite Difference Methods*, Springer, 1998.

[8] Leveque, R. J.,*Numerical Methods for Conservation Laws*, Lectures in Mathematics, ETH-Zurich, Birkhäuser-Verlag, Basel, 1990.

NOTE: The students opting for M.Sc. (Applied Mathematics) should take this course and follow it up with any three out of six courses (AM576-AM581) in their fourth Semester.

16 Mathematical Methods (Code: AM503/MM503)

INTEGRAL TRANSFORMS:

Laplace transforms: Definitions, properties, Laplace transforms of some elementary functions, convolution theorem, inverse Laplace transformation, applications.

Fourier transforms: Definitions, properties, Fourier transforms of some elementary functions, convolution theorems, Fourier transform as a limit of Fourier Series.

INTEGRAL EQUATIONS:

Volterra integral equations: Basic concepts, relationship between linear differential equations and Volterra integral equations - resolvent kernel of Volterra integral equations, solution of integral equations by resolvent kernel, the method of successive approximations, convolution type equations, solution of integro- differential equations with the aid of Laplace transformation.

Fredholm integral equations: Fredholm equations of the second kind, fundamentals, iterated kernels, constructing the resolvent kernel with the aid of iterated kernels, integral equations with degenerate kernels, characteristic numbers and eigenfunctions, solution of homogeneous integral equations with degenerate kernel, nonhomogeneous symmetric equations, Fredholm alternative.

Fredholm operator as a compact operator on $L^2[a, b]$; properties of characteristic numbers and eigenfunctions for symmetric kernels; application of spectral theorem and series solution in the case of symmetric kernels.

CALCULUS OF VARIATIONS:

Extrema of functionals: The variation of a functional and its properties, Euler's equation, field of extremals, sufficient and necessary conditions for the extremum of a functional both for weak and strong extrema; Legendre and Weierstrass theorems, Hilbert invariant integral theorem, conditional extremum, moving boundary problems, discontinuous problems, one sided variations, Ritz method.

References

[1] Brunt, Bruce van, *The Calculus of Variations* , Springer-Verlag, New York, 2004.

[2] Sneddon I. N., *The Use of Integral Transforms* Tata McGraw Hill, 1972.

[3] Spiegel,Murray R., *Schaum's Outline of Laplace Transforms*, Schaum's Outline Series, 1965.

[4] Gelfand, I. M. and Fomin, S. V., *Calculus of Variations*, revised English edition translated and edited by Richard A. Silverman, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963.

[5] Krasnov, M. L.; Makarenko, G. I. and Kiselëv, A. I., *Problems and Exercises in the Calculus of Variations*, translated from the Russian by George Yankovsky, 1975.

[6] Krasnov, M. L.; Makarenko, G. I. and Kiselëv, A. I., *Problems and Exercises in Integral Equations*, translated from the Russian by George Yankovsky, 1975.

[7] Kanwal Ram P., *Linear Integral Equations*, second edition, Birkhäuser Boston, Inc., Boston, MA, 1997.

[8] Pipkin, Allen C., *A Course on Integral Equations*, Texts in Applied Mathematics, 9, Springer-Verlag, New York, 1991.

[9] Gibbons, M. M., *A Primer on the Calculus of Variations and Optimal Control Theory*, Volume-50, AMS, 2009.

IV Semester (Math./Appl.Math.)

17 Representation Theory (Code: MM562)

Modules basic theory; tensor product of modules, tensor algebra, symmetric algebra, exterior algebra; Wedderburn Artin theory; group rings.

Definition of a representation of a finite group, irreducible representation, equivalent representations, representation as a group ring module.

Character theory, irreducible characters as an orthonormal basis of class functions.

Induced representations, tensor of representations; Mackey theory and applications.

Algebraic integers, Burnside's theorem.

Artin's theorem and Brauer's theorem.

References

[1] Serre, J. P., *Linear Representations of Finite Groups*, Springer, 1977.

[2] Jacobson, N., *Lectures in Abstract Algebra, 3*, East West Press,1966.

[3] Artin, M., *Algebra*, Prentice Hall of India, 1994.

18 Lie Algebras (Code: MM573/AM573)

Definition of Lie algebras, classical examples, ideals, standard isomorphism theorems, nilpotent Lie algebras, solvable Lie algebras, simple Lie algebras, Engel's theorem, Lie's theorem, Jordan decomposition, Cartan's criterion for solvability, Cartan-Killing form, semisimplicity, SL(2) representations, Cartan subalgebras leading to root systems, study of simple root systems, Weyl group, simple root systems, Dynkin diagrams, classification of simple root systems, universal enveloping algebra, statement of PBW theorems, roots and weights calculation in classical set up, statement of Serre's theorem, definition and basic properties of Verma modules, statement of classification of finite dimensional representations of simple Lie algebras.

References

[1] Humphreys, J., *Introduction to Lie Algebras and Representation Theory*, GTM 9, Springer-Verlag, 1972.

[2] Serre, Jean-Pierre, *Complex Semisimple Lie Algebras*, Springer Monographs, 2001.

19 Commutative Algebra (Code: MM575)

Recall basics of commutative rings, prime ideals, maximal ideals, prime spectrum, primary ideals, nilradical, Jacobson radical. Ring of fractions, going up/going down theorems, primary decomposition, Hilbert's nullstellansatz, Noether normalization lemma, Krull dimension, Chevally dimension, regular rings, DVRs, normal integral domain.

References

[1] Atiyah, M. F., and Macdonald, I. G., *Introduction to Commutative Algebra*, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1969.

[2] Zariski, O. and Samuel, P., *Commutative Algebra, Vol. I*, GTM No.28, Springer-Verlag, New York, 1958.

[3] Matsumura, Hideyuki, *Commutative Algebra*, second edition, Mathematics Lecture Note Series, 56, Benjamin/Cummings Publishing Co., Inc., Reading, Mass., 1980.

[4] Gopalakrishnan, N. S., *Commutative Algebra*, Oxonian press, 1984.

[5] Reid, M, *Undergraduate Commutative Algebra*, Cambridge University Press, 1995.

20 Algebraic Geometry (Code:MM578)

Commutative algebra: Localization, dimension theorem for Noetherian rings (without proof), Rings of dimension 0, 1, normal Noetherian rings of dimension 1 (i.e., Dedekind domains), normalization lemma and Hilbert's null stellensatz (without proof), transcendental extensions and Luroth's theorem.

Geometric concepts: Prime spectrum of a commutative ring with Zariski topology, irreducible algebraic sets and affine algebraic varieties, criterion for connectedness of affine algebraic sets; Noetherian topological spaces, principle of Noetherian induction and application to algebraic varieties (decomposing an algebraic variety into a finite union of irreducible components); projective spectrum of a polynomial ring and projective varieties.

Affine plane curves: Classification of algebraic subsets of the plane, degree of a plane curve, intersection of curves (via dimension theorem and also via elimination theory) and weak form of Bezout's theorem; regular and rational functions on a curve.

Rational curves: Rational and non-rational curves, conics, a characterization of rational curves in terms of the field of rational functions on the curve, birational isomorphisms.

Projective plane curves: Projective completion of an affine curve, homogenization and dehomogenization, resultant of homogeneous polynomials.

Analysis of singularities: Order of contact of a line with a curve at a point, multiplicity of a point, smooth and multiple points, tangent space and tangent cone at a point, simple and multiple tangents, ordinary multiple points, finiteness of the singular locus of a curve; characterization of smooth points and r-fold points, Sezout's theorem.

Classification of Curves (brief discussion): Topology of a non-singular irreducible curve as a compact Riemann surface, genus of a curve, formula for the genus of curve with at most ordinary singularities, rational curves as curves of genus 0, elliptic curves.

Elliptic functions and Weierstrass' p-function, elliptic curve as an abelian variety (geometric and analytic versions) and the isomorphism classes of elliptic curves as the upper half plane modulo the modular group, etc.

References

[1] Miles, Reid, *Undergraduate Algebraic Geometry*, Student Text Books, London Mathematical Society Student Texts (Book 12), Cambridge University Press, 1989.

[2] Walker, R. J., *Algebraic Curves*, Springer-Verlag, Berlin-New York, 1978.

[3] Hartshorne, R., *Algebraic Geometry*, Issue 52 of Graduate Texts in Mathematics, Lecture notes in mathematics, Volume 687, Springer, 1977.

[4] Shafarevich, I. R., *Basic Algebraic Geometry*, GTM, Springer, second revised and expanded edition, 1994.

[5] Atiyah, M. F. and Macdonald, I. G., *Introduction to Commutative Algebra*, Addison-Wesley Publishing Co., 1969.

[6] Fulton, W., Algebraic Curves, *An Introduction to Algebraic Geometry*, Notes written with the collaboration of Richard Weiss, Mathematics Lecture Notes Series, W. A. Benjamin, Inc., New York-Amsterdam, 1969.

21 Complex Algebraic Geometry (Code: MM580)

The course will introduce students to algebraic geometry over the complex numbers beginning with the analytic theory, serving as an invitation to algebraic geometry.

Plane curves, morphism and rational maps, complex varieties, analytic topology, Zariski topology, compact orientable manifolds, compact Riemann surfaces, projective algebraic curves, Bezout's Theorem, differential forms, integration, divisors on Riemann surfaces and linear equivalence, genus of a compact Riemann surface, canonical divisors on a Riemann surface, divisors of poles and zeroes, linear systems, Riemann-Roch theorem, Serre duality, and applications.

References

[1] Miranda, Rick, *Algebraic Curves and Riemann Surfaces*, Graduate Studies in Mathematics 5, AMS, 1995.

[2] Forster, Otto, *Lectures on Riemann Surfaces*, Springer, 1999.

[3] Griffiths, Phillip A., *Introduction to Algebraic Curves* Translations of Mathematical Monographs (Book 76), American Mathematical Society, 1989.

[4] Kirwan, Frances, *Complex Algebraic Curves*, London Mathematical Society Student Texts, 23, Cambridge University Press, 1992.

[5] Fulton, William, *Algebraic Curves. An Introduction to Algebraic Geometry*, reprint of 1969 original, Addison-Wesley, 1989.

22 Elliptic Curves (Code: MM583)

A little of Projective Geometry, statement without proof of Bezout's theorem, reduction modulo p.

Weierstrass Normal form, Group law on elliptic curves, points of finite order, discriminant and the Nagell-Lutz theorem.

The group of rational points of an elliptic curve; proof of Mordell's theorem for curves with a rational point of order 2.

Curves over finite fields, Gauss's theorem on the curve $X^3 + Y^3 = 1$, Lenstra's algorithm for using elliptic curves in Cryptology.

Elliptic curves with complex multiplication, Galois representations, abelian extensions of $Q(i)$.

(Optional) Integer points on elliptic curves and Thue's theorem.

References

[1] Silverman, J. and Tate, J., *Rational Points on Elliptic Curves*, Undergraduate Texts in Mathematics, Springer-Verlag, New York, 1992.

[2] Silverman, Joseph H., *The Arithmetic of Elliptic Curves* (Graduate Texts in Mathematics), second edition, 2009.

[3] Milne, J. S., *Elliptic Curves*, BookSurge Publishing, 2006.

23 Banach Algebras (Code: MM572/AM572)

Preliminaries: Banach spaces, weak and weak-topologies on Banach spaces, Banach space valued function and their derivatives, holomorphic functions, Banach space values, measures and integration.

Banach algebras : Definition, homomorphism, spectrum, basic properties of spectra, Gelfand-Mazur theorem, spectral mapping theorem, group of invertible elements.

Commutative Banach algebras and Gelfand theory: Ideals, maximal ideals and homomorphism, semi-simple Banach algebra, Gelfand topology, Gelfand transform, involutions. Banach *-algebras, Gelfand-Naimark theorem, applications to non-commutative Banach algebras, positive functions.

Operators on Hilbert spaces: Commutativity theorem, resolution of the identity, spectral theorem, a characterization of Banach * - algebras.

References

[1] Allan, G. R.,*Introduction to Banach Spaces and Algebras*, Oxford Graduate Texts in Mathematics 20, Oxford University Press, 2011.

[2] Douglas, R. G., *Banach Algebra Techniques in Operator Theory*, Graduate Texts Mathematics 179, Springer-Verlag, 1998.

[3] Fillmore, P. A., *A User's Guide to Operator Algebras*, Canadian Mathematical Society Series of Monographs and Advanced Texts, John Wiley & Sons, 1996.

[4] Rudin, W., *Functional Analysis*, second edition., McGraw-Hill, 1991.

24 Complex Analysis - II (Code: MM571/AM571)

Space of continuous functions, the space of analytic functions, the space of meromorphic functions, Riemann-mapping theorem, Mittag Leffler's theorem, analytic continuation along paths, Mono-dromy theorem, Picard's theorem, harmonic functions, entire functions, normal families, elliptic functions.

References

[1] Conway, J. B., *Functions of One Complex Variable*, second edition, Springer-Verlag Berlin and Heidelberg, 2001.

[2] Ahlfors, Lars V., *Complex analysis. An Introduction to the Theory of Analytic Functions of One Complex Variable*, third edition, International Series in Pure and Applied Mathematics, McGraw-Hill Book Co., New York, 1978.

[3] Rudin, Walter, *Real and Complex Analysis*, third edition, McGraw-Hill Book Co., New York, 1987.

25 Graph Theory and Algorithms (Code: MM581/AM581)

Introduction.

Paths and Circuits: Euler graphs, Hamiltonian paths and circuits.

Trees: Rooted and binary trees, spanning trees, fundamental circuits, spanning trees in a weighted graph.

Cut-sets: Fundamental circuits and cut-sets, network flows, 1-isomorphism, 2-isomorphism.

Planar Graphs: Kuratowski two graphs, detection of planarity, geometric dual, thickness and crossing.

Matrix representation of graphs.

Colouring and covering: chromatic number, four colour problem.

Directed graphs: Digraphs and binary relations, Euler digraphs.

Algorithms on Graphs: Minimum cost spanning trees, depth first search, strong connectivity, path finding problems, transitive closure algorithm, shortest path algorithm, path problems and matrix multiplication, single source problems.

References

[1] Harary, F., *Graph Theory*, Addison-Wesley Publishing Co., London, 1969.

[2] Deo, Narsingh, *Graph Theory with Applications to Engineering and Computer Science*, Prentice-Hall Series in Automatic Computation, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1974..

[3] Aho, A. V.; Hopcroft, J. E. and Ullman, J. D., *The Design and Analysis of Computer Algorithms*, second printing, Addison-Wesley Series in Computer Science and Information Processing, Addison-Wesley Publishing Co., 1975.

[4] Gibbons, A., *Algorithmic Graph Theory* Cambridge University Press, Cambridge, 1985.

26 Topology - II (Code: MM577/AM577)

Fundamental groups and homotopy theory: homotopic mappings, contractible spaces, essential and inessential maps, homotopically equivalent spaces, fundamental group of space, examples, homotopy groups $\pi_n(X)$, $n \geq 1$.

Simplicial theory: Simplicial complexes, bary centric subdivision, simplicial maps, approximation theorem, fundamental group of a simplicial complex.

Covering space theory: Covering Spaces, examples, properties of covering spaces, relation between fundamental group of covering space and its base and tower correspondence, universal covering space construction.

Simplicial and singular homology theory:

(a) Oriented complexes, chains, cycles and boundary operator. Homology groups, betti numbers and torsion coefficients, zero dimensional homology, Euler Poincaré formula, simplicial maps and induced homeomorphisms, chain complexes and chain maps and induced homeomorphisms, cone complexes.

(b) Singular homology theory, exact sequences, excision, Mayer-Vietoris sequence etc.

(c) Cech homology theory: Axiomatic homology theory of Eilenberg, Steenrod and its properties, Cech homology and properties.

References

[1] Bredon, G. E., *Topology and Geometry*, Graduate Texts in Mathematics 139, Springer-Verlag, 1997.

[2] Hatcher, A., *Algebraic Topology*, Cambridge University Press, 2002.

[3] Hocking, J. G. and Young, G. S., *Topology*, second edition, Dover Publications, New York, 1988.

[4] Greenberg, M. J. and Harper, J. R.,*Algebraic Topology: A First Course*, Mathematics Lecture Note Series 58, Benjamin/Cummings Publishing Co., 1981.

[5] Massey, W. S., *A Basic Course in Algebraic Topology*, Graduate Texts in Mathematics 127, Springer-Verlag, 1991.

[6] Maunder, C. R. F., *Algebraic Topology*, Cambridge University Press, 1980.

27 Algebraic Topology - (Code: MM594/AM594)

Homotopy theory: Homotopy; retraction and deformation; suspension; mapping cylinder; fundamental group; the Van Kampen theorem; covering spaces; homotopy lifting property; relations with the fundamental group; lifting of maps; universal coverings; automorphisms of a covering; introduction to the category theory, the fundamental group as an example of functor, the basic definitions regarding higher homotopy groups. 2. Simplicial homology and singular homology: Simplicial homology groups; singular homology groups; homomorphisms induced by homotopic maps; Sperner's lemma; Brouwer's fixed point theorem; Jordan curve theorem; the invaraince of domain.

References

[1] Bredon, G. E., *Topology and Geometry*, Graduate Texts in Mathematics 139, Springer-Verlag, 1997.

[2] Hatcher, A., *Algebraic Topology*, Cambridge University Press, 2002.

[3] Hocking, J. G. and Young, G. S., *Topology*, second edition, Dover Publications, New York, 1988.

[4] Greenberg, M. J. and Harper, J. R.,*Algebraic Topology: A First Course*, Mathematics Lecture Note Series 58, Benjamin/Cummings Publishing Co., 1981.

[5] Massey, W. S., *A Basic Course in Algebraic Topology*, Graduate Texts in Mathematics 127, Springer-Verlag, 1991.

[6] Maunder, C. R. F., *Algebraic Topology*, Cambridge University Press, 1980.

[7] Narasimhan, M. S., Ramanan, S., Sridharan, R., Varadarajan, K. *Algebraic topology*, TIFR, Bombay, 1964.

[8] Spanier, E. H. *Algebraic Topology*, McGraw Hill, 1967.

28 Advanced Functional Analysis (Code :MM595/AM595)

Weak and weak* topology, convex sets and extreme points, convexity and smoothness of the norm, bases in Banach spaces, basic spectral theory of operators, advanced results about well-studied classes of operators such as compact operators and normal operators.

References

[1] Abramovich, Y. A., Aliprantis, C. D., *An Invitation to Operator Theory*, Graduate Studies in Mathematics, 50, American Mathematical Society, Providence, RI, 2002.

[2] Albiac, F., Kalton, N., *Topics in Banach Space Theory*, Second edition, Graduate Texts in Mathematics, 233, Springer, 2016.

[3] Conway, J. B., *A Course in Operator Theory*, Graduate Studies in Mathematics, 21, American Mathematical Society, Providence, RI, 2000.

[4] Fabian, M., Habala, P., Hájek, P., Montesinos, V., Zizler, V., *Banach Space Theory*, CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC, Springer, New York, 2011.

[5] Harte, R., *Spectral Mapping Theorems. A Bluffer's Guide*, Springer Briefs in Mathematics, Springer, 2014.

[6] Kubrusly, C. S., *Spectral Theory of Operators on Hilbert Spaces*, Birkhäuser/Springer, New York, 2012.

[7] Lax, P., *Functional Analysis*, Pure and Applied Mathematics (New York), Wiley-Interscience [John Wiley & Sons], New York, 2002.

29 Discrete Dynamical Systems (Code: MM591/AM591)

Phase portraits, periodic points and stable sets, Sarkovskii's theorem, hyperbolic, attracting and repelling periodic points.

Families of dynamical systems, bifurcation, topological conjugacy.

The logistic function, Cantor sets and chaos, period-doubling cascade.

Symbolic dynamics.

Newton's method.

Numerical solutions of differential equations.

Complex dynamics, quadratic family, Julia sets, Mandelbrot set.

Topological entropy, attractors and fractals, theory of chaotic dynamical systems.

References

[1] Holmgren, R. M., *A First Course in Discrete Dynamical Systems*, Springer-Verlag, 1996.

[2] Devaney, Robert L., *Introduction to Chaotic Dynamical Systems*, Addison-Wesley, 1989.

[3] Brin, M. and Stuck, G., *Introduction to Dynamical Systems*, Cambridge University Press, 2002.

30 Dynamical Systems (Code: MM574/AM574)

Review of linear systems.

Dynamical systems and vector field, fundamental theorem, existence and uniqueness, continuity of solutions in initial conditions, extending solutions, global solutions, flow of a differential equation.

Stability of equilibrium, nonlinear sinks, stability, Liapunov functions, gradient systems.

The Poincaré-Bendixson theorem and applications.

Introduction to discrete dynamical systems.

References

[1] Hirsch, Morris W. and Smale, Stephen, *Differential Equations, Dynamical Systems, and Linear Algebra* Pure and Applied Mathematics, Vol. 60. Academic Press (A subsidiary of Harcourt Brace Jovanovich, Publishers), New York-London, 1974.

[2] Holmgren, R. A., *A First Course in Discrete Dynamics*, Springer-Verlag, 1994

31 Ergodic Theory (Code: MM579/AM579)

Poincaré's recurrence theorem, Hopf's maximal ergodic theorem, Birkhoff's individual ergodic Theorem, von Neumann's mean ergodic theorem.

Ergodicity, mixing, eigenvalues, discrete spectrum theorem, ergodic automorphisms of compact groups, conjugacy, entropy.

References

[1] Halmos, P. R., *Lectures on Ergodic Theory*, American Mathematical Society, 2006.

[2] Nadkarni, M. G., *Basic Ergodic Theory*, TRIM series, Hindusthan Book Agency, third edition, 2013.

32 Harmonic Analysis (Code: MM592/AM592)

Fourier analysis : Fourier series, pointwise and uniform converges of Fourier series, Fourier transforms, Riemann-Lebesgue lemma, inversion theorem, Parseval identity, Plancharel theorem.

Topological groups: Definition, Basic properties, subgroups, quotient groups, locally compact topological groups, examples.

Haar integral: Left and right invariant Haar measure, its existence and uniqueness on locally compact topological group, Examples of Haar measures.

Compact groups: Representations of compact groups, Peter-Weyl theorem Examples such as $SU(2)$ and $SO(3)$.

Elements of Banach algebras: Analytic properties of functions from $\mathbb{C}$ to Banach algebras, spectrum and its compactness, commutative Banach algebras, Maximal ideal space, Gelfand topology.

Generalization of Fourier transform : Fourier transform on $L'(G)$ and $L(G)$ (G being a locally compact topological group) Positive definite functions, Bochner characterization, inversion formula, Plancheral theorem, Pontrjagin Duality theorem.

References

[1] Folland, G. B., *A Course in Abstract Harmonic Analysis*, CRC Press, 1995.

[2] Deitmar, Anton, *A First Course in Harmonic Analysis*, second edition, Springer, 2002.

[3] Katznelson, Yitzhak, *An Introduction to Harmonic Analysis*, third edition, CUP, 2002.

[4] Helson, H., *Harmonic Analysis*, Addison Wesley, 1983.

[5] de Vito, C., *Harmonic Analysis - A Gentle Introduction*, Jones & Bartlett, 2007.

33 Number Theory (Code: MM576)

Finite fields, equations over finite fields, Chevalley's theorem, law of quadratic reciprocity.

Construction of the p-adic field $\mathbb{Q}_p$, range $\mathbb{Z}_{pj}$, properties of $\mathbb{Z}_p$. P-adic equations, group structure of $\mathbb{Q}_p^*$.

Hilbert symbol over $\mathbb{Q}_p$, $\mathbb{R}$, $\mathbb{Q}$, product formula and application.

Quadratic forms over $\mathbb{Q}_p$, $\mathbb{R}$ and $\mathbb{Q}$, their classification and invariants theorem of Hasse and Minkowski. Gauss's theorem. (Or, Integral quadratic forms with discriminant ± 1).

Dirichlet's theorem on primes in an arithmetic progression.

Introductory Modular forms.

References

[1] Serre, J. P., *A course in Arithmetic*, Springer GTM 42, 1977.

[2] Flath, J., *Introduction to Number Theory*, John Wiley and Sons, 1989.

34 Number Theory and Cryptography (Code: MM593)

Divisibility and Euclidean algorithm, congruences, applications to factoring.

Finite fields, Legendre symbol and quadratic reciprocity, Jacobi symbol.

Cryptosystems, diagraph transformations and enciphering matrices, RSA Cryptosystem.

Primality and factoring, Pseudoprimes, Carmichael no, primality tests, pseudoprimes, Monte Carlo method, Fermat factorization, factor base, implication for RSA, continued fraction method.

Elliptic curves - basic facts, elliptic curves over $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Q}$, finite fields. Hasse's theorem (without proof), Weil's conjectures (without proof), elliptic curve cryptosystems, elliptic curve factorization - Lenstra's method.

References

[1] Koblitz, Neal, *A Course in Number Theory and Cryptography*, Graduate Texts in Mathematics, Springer, 1987.

[2] Rosen, M. and Ireland, K., *A Classical Introduction to Number Theory*, Graduate Texts in Mathematics, Springer, 1982.

[3] Bressoud, David, *Factorization and Primality Testing*, Undergraduate Texts in Mathematics, Springer, 1989.

35 Algebraic Number Theory (Code: MM585)

Classification theorem of nondiscrete, locally compact topological fields. (optimal)

Discrete valuation rings and their basic properties.

Algebraic number fields; Dedekind domains, class group, finiteness of class number, structure of unit group.

Quadratic extensions of $\mathbb{Q}$, quadratic reciprocity, class number formula.

References

[1] Weil, A., *Basic Number Theory*, Springer, Classics in mathematics, 1973.

[2] TIFR Mathematical Pamphlet: Algebraic Number Theory (available on TIFR website)

[3] Artin, M., *Algebra*, Prentice Hall of India, 1991.

[4] Serre, J.P., *Local Fields*, Springer, Graduate Texts in Math, 1995.

36 Number Theory and Equations over Finite Fields (Code : MM584/AM584)

Prime Numbers and Unique Factorization, Primes in Arithmetic Progressions, Euclids Alghorithm, Wilsons Theorem, Linear congruence; $ax \equiv b(\text{mod } n)$, Sums of Two Squares, Chinese Remainder Theorem, Euler's Theorem.

Primitive roots modulo n, Structure of $U(\mathbb{Z}/n\mathbb{Z})$, The equation $x^n \equiv a(\text{mod } m)$ (n^{th} Power residues), The ring of Gaussian integers $\mathbb{Z}[i]$, Integral Binary Quadratic forms $aX^2 + bXY + cY^2$, Quadratic Reciprocity Laws: Legender Symbol and a Gauss Sum.

Finite Fields, Gauss and Jacobi Sums, Equations over Finite Fields: Chevalley-Warning Theorem, Quadratic Forms over finite fields and their reduction to the equation $a_1x_1^{l_1} + a_2x_2^{l_2} + \cdots + a_rx_r^{l_r} = b$ over $\mathbb{F}_q$.

Construction of p-adic numbers, ring of p-adic integers, some appliations.

References

[1] Stilwell, J., *Elements of Number Theory*, Sprinter UTM 2003.

[2] Gareth A. Jones and Mary Jones J., *Elementary Number Theory*, Springer SUMS 2005.

[3] Ireland K. and Rosen M., *A Classical Introduction to Modern Number Theory*, Springer GTM 2004.

[4] Lidl R. and Niedrreiter H., *Finite Fields, Encyclopedia of Mathematics and its Applications* 20, Cambridge 1997.

[5] Flath D.E., *Introduction to Number Theory*, John Wiley & Sons 1989.

[6] Serre J.P., *A Course in Arithmetic*, Springer GTM 42, 1977.

37 Classical Mechanics (Code: AM570/MM570)

Curvilinear co-ordinates : Cylindrical and spherical polar co-ordinates.

Mechanics of a particle, mechanics of a system of particles, types of constraints, d'Alembert's principle, Lagrange's equations, Lagrangian formulation in generalized co-ordinates, variational principles, Hamilton's principle of least action, derivation of Lagrange's equations from Hamilton's principle, Legendre transformation, Hamiltonian, canonical equations, cyclic co-ordinates, Routh's procedure, generating functions, Poisson brackets, Liouville's theorem, infinitesimal canonical transformations, conservation theorems and angular momentum relations using Poisson brackets, Hamilton-Jacobi equations, Hamilton's Principal Function, example of Harmonic oscillator, Hamilton-Jacobi equation for Hamilton's Characteristic Function, action-angle variables in systems of one degree of freedom.

Central force problem: Equations of motion and first integrals, equivalent one dimensional problem, classification of orbits; Kepler's problem: inverse square law of force.

Moving frames of reference: Non-inertial frames of reference, rate of change of a vector in a rotating frame, applications to particle kinetics, motion relative to earth, effects of Coriolis.

Two dimensional problems in rigid body dynamics, examples.

Kinematics of rigid body motion: Euler angles, Euler's theorem on the motion of a rigid body, infinitesimal rotations.

The rigid body equations of motion : Angular momentum and kinetic energy of motion, inertia tensor and moment of inertia, inertial ellipsoid, the eigenvalues of the inertia tensor and the principal axis transformation, Euler's dynamical equations of motion under no external forces, torque-free motion of a rigid body, heavy symmetrical top with one point fixed.

References

[1] Goldstein, H., Poole, C. P. and Safko, J., *Classical Mechanics*, third edition, Pearson, 2011.

[2] Chorlton, F., *Textbook of Dynamics*, second edition, Ellis Horwood Series: Mathematics and its Applications, Halsted Press (John Wiley & Sons, Inc.), New York, 1983.

[3] Marion, J. B. and Thornton, S. T., *Classical Dynamics of Particles and Systems*, third edition, Harcourt Brace Jovanovich, 1988.

[4] Scheck, Florian, *Mechanics. From Newton's Laws to deterministic Chaos*, fifth edition, Graduate Texts in Physics, Springer, Heidelberg, 2010.

[5] Marsden, Jerrold E. and Ratiu, Tudor S., *Introduction to Mechanics and Symmetry*, A Basic Exposition of Classical Mechanical Systems, Texts in Applied Mathematics, 17, Springer-Verlag, New York, 1994.

[6] José, Jorge V. and Saletan, E. G., *Classical Dynamics, A Contemporary Approach*, Cambridge University Press, 1998.

38 Fluid Dynamics (Code: AM569/MM569)

Continuum hypothesis, forces acting on a fluid, stress tensor, analysis of relative motion in the neighborhood of a point, Euler's theorem, equation of continuity, Reynolds transport theorem, conservation of mass, material surface, momentum equation.

Stream lines, Bernoulli's theorem, energy equation, circulation, Kelvin's circulation theorem, vorticity, Lagrange's theorem on permanence of vorticity, two dimensional irrotational flow of an incompressible fluid, Milne-Thomson circle theorem, Blasius' theorem, flow past an airfoil, the Joukowski transformation, theorem of Joukowski and Kutta.

Axisymmetric flows, Stokes stream function, Butler's sphere theorem, flows due to source, doublet, uniform flow past a sphere, irrotational three dimensional flow, Weiss' sphere theorem.

Constitutive equations for incompressible fluids, derivation of Navier-Stokes equations, unidirectional flows, Poiseuille flow, Couette flow, Stokes first and second problems, stagnation point flows, dynamical similarity and Reynolds number.

Flows at low Reynolds number, axisymmetric flow of a viscous fluid, uniform flow past a sphere at low Reynolds number, torque and drag on a sphere due to a uniform flow,

Prandtl model for boundary layer, boundary layer equation, solution for a flow past a plate.

References

[1] Batchelor, G. K., *An Introduction to Fluid Mechanics*, Cambridge University Press, 1993.

[2] Happel, J. and Brenner, H., *Low Reynolds Number Hydrodynamics with Special Applications to Particulate Media*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1965.

[3] Schlichting, H. and Gersten, K., *Boundary-Layer Theory*, with contributions by Egon Krause and Herbert Oertel, Jr. translated from the ninth German edition by Katherine Mayes, eighth revised and enlarged edition, Springer-Verlag, Berlin, 2000.

[4] Landau, L. D. and Lifshitz, E. M., *Fluid Mechanics*, Pergamon Press, London-Paris-Frankfurt; Addison-Wesley Publishing Co., Inc.,1959.

[5] Kambe, T., *Elementary Fluid Mechanics*, World Scientific Publishing Co. Pvt. Ltd., Hackensack, NJ, 2007.

[6] O'Neill, M. E. and Chorlton, F., *Ideal and Incompressible Fluid Dynamics*, Ellis Horwood Series: Mathematics and its Applications. Ellis Horwood Ltd., Chichester; Halsted Press (John Wiley & Sons, Inc.), New York, 1986.

[7] Chorin, A. J. and Marsden, J. E., *A Mathematical Introduction to Fluid Mechanics*, third edition, Texts in Applied Mathematics, 4, Springer-Verlag, New York, 1993.

39 Continuum Mechanics (Code: AM586/MM586)

Cartesian tensors.

Description of continua and kinematics.

Forces in a continuum.

The polar decomposition theorem.

Continuum deformation.

Geometrical restrictions on the form of constitutive equations.

Constitutive equations for fluid, elastic and thermo-elastic materials.

Shear flow solutions of Reiner-Rivlin fluids.

Some solutions of the Navier-Stokes equations.

General theorems in inviscid hydrodynamics.

References

[1] Hunter, S. C., *Mechanics of continuous Media*, Mathematics and its Applications. Ellis Horwood Ltd., Chichester Halsted Press, (John Wiley and Sons Inc.), New York-London-Sydney, 1976.

[2] Chadwick, P., *Continuum Mechanics: Concise Theory and Problems*, Dover, second edition, Dover Publications, Inc., Mineola, NY, 1999.

[3] Lai, W. M., Rubin, D. and Krempl, E., *Introduction to Continuum Mechanics*, third edition, Butterworth Heinemann Ltd., 1993.

[4] Sedov, L. I., *A Course in Continuum Mechanics*, Wolters-Noordhoff, Groningen, 1971.

[5] Narasimhan, M. N. L., *Principles of Continuum Mechanics*, Wiley, New York, 1993.

40 Ordinary Differential Equations - II (Code: AM587/MM587)

Test functions, null sequences, distributions and its properties, derivatives of distributions, weak solutions, adjoint operators, fundamental solutions of an operator, Fourier transforms for distributions, Heisenberg uncertainty principle.

Existence and uniqueness on solutions of boundary value problems in O.D.E.

Green's function for O.D.E. of all orders.

Poincaré-Bendixon theorem with proof.

Peano's existence theorem for a system of O.D.E.

Picard's theorem for a system of O.D.E.

Bifurcation points, saddle node bifurcation, pitch fork bifurcation, Hopf bifurcation.

Stability analysis in higher dimensions.

References

[1] Stakgold, Ivar, *Green's Functions and Boundary Value Problems*, second edition, Pure and Applied Mathematics (New York), A Wiley-Interscience Publication, John Wiley & Sons, Inc., New York, 1998.

[2] Grimshaw, R., *Nonlinear Ordinary Differential Equations*, Applied Mathematics and Engineering Science Texts, CRC Press, Boca Raton, FL, 1993.

[3] Hirsch, Morris W. and Smale, Stephen, *Differential Equations, Dynamical Systems, and Linear Algebra*, Pure and Applied Mathematics, Vol. 60, Academic Press (A subsidiary of Harcourt Brace Jovanovich, Publishers), New York-London, 1974.

[4] Jordan, D. W.; Smith, P., *Nonlinear Ordinary Differential Equations*, second edition, Oxford Applied Mathematics and Computing Science Series, The Clarendon Press, Oxford University Press, New York, 1987.

[5] King, A. C.; Billingham, J. and Otto, S. R., *Differential Equations: Linear, Nonlinear, Ordinary, Partial*, Cambridge University Press, Cambridge, 2003.

[6] Hirsch, M. W; Smale, S. and Devaney, R. L., *Differential Equations and Dynamical Systems and an Introduction to Chaos*, Academic Press, Elsevier, 2004.

41 Partial Differential Equations - II (Code: AM588/MM588)

UNIT - 1 :

Perturbation theory for matrices: operator norm of a matrix, diagonal dominance, condition number, relative error.

Finite Difference Methods(FDM): Consistency, stability, accuracy of FDM, Lax equivalence theorem, Von Neumann (L^2) stability analysis, CFL condition, upwind schemes, Godunov scheme, Lax-Friedrichs' scheme, Lax-Wendroff scheme, examples of higher order schemes for transport equation; Crank-Nicolson scheme, θ-scheme for the heat equation and their stability analysis.

Convergence of a FDM scheme for Poisson equation in bounded domain, approximating operators, Kantorovich theorem.

UNIT - 2 :

Theory of scalar conservation laws:
Motivation and examples of conservation laws, finite time blow up of smooth solution, notion of weak solution, R-H jump condition, shock and rarefaction waves, entropy conditions of Lax, Oleinik, Kruzkov, Lax-Oleinik formula, existence and uniqueness of the entropy solution, Riemann problem for convex, non-convex (general) flux functions.

UNIT - 3 :

Numerical study of Scalar Conservation Laws :
Schemes in the conservative form, numerical flux, Lax-Wendroff theorem, Godunov scheme, Lax-Friedrichs' scheme, Murman-Roe scheme, Engquist-Osher scheme, Lax-Wendroff scheme, monotone, monotonicity preserving and T.V.D. schemes, Harten theorem; schemes satisfying entropy condition.

References

[1] Evans, L. C., *Partial Differential Equations*, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society, Providence, RI, 1998.

[2] Godlewski, E. and Raviart, P. A., *Hyperbolic Systems of Conservation Laws*, Mathématiques & Applications, 3/4, Ellipses, Paris, 1991.

[3] LeVeque, R. J., *Numerical Methods for Conservation Laws*, Lectures in Mathematics, ETH, Zürich, Birkhäuser, 1992.

[4] LeVeque, R. J., *Finite Volume Methods for Hyperbolic Problems*, Cambridge University Press, 2002.

[5] Smoller, J., *Shock Waves and Reaction-Diffusion Equations*, Springer-Verlag, 1983.

42 Differential Geometry (Code: AM582/MM582)

Local curve theory: Serret-Frenet formulation, fundamental existence theorem of space curves.

Plane curves and their global theory: Rotation index, convex curves, isoperimetric inequality, Four vertex theorem.

Local surface theory: First fundamental form and arc length, normal curvature, geodesic curvature and Gauss formulae, Geodesics, parallel vector fields along a curve and parallelism, the second fundamental form and the Weingarten map, principal, Gaussian, mean and normal curvatures, Riemannian curvature and Gauss's theorem Egregium, isometries and fundamental theorem of surfaces.

Global theory of surfaces: Geodesic coordinate patches, Gauss-Bonnet formula and Euler characteristic, index of a vector field, surfaces of constant curvature.

Elements of Riemannian geometry: Concept of manifold, tensors (algebraic and analytic), covariant differentiation, symmetric properties of curvature tensor, notion of affine connection, Christoffel symbols; Riemannian metric and its associated affine connection, geodesic and normal coordinates (if time permits).

References

[1] Millman, R. S. and Parker, G. D., *Elements of Differential Geometry*, Prentice Hall Inc., 1977.

[2] Klingen, Berg W., *A Course in Differential Geometry*, translated from the German by David Hoffman, Graduate Texts in Mathematics, Vol. 51. Springer-Verlag, New York-Heidelberg, 1978.

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Lie groups of transformations.

Infinitesimal transformations.

Infinitesimal generators.

Extended transformations.

Invariance of an ordinary differential equation.

Canonical coordinates.

Determination of first order ordinary differential equations invariant under a given group.

Invariance of O.D.E. under multi-parameter groups.

Invariance of P.D.E.

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44 Nonlinear Programming (Code: MM590/AM590)

Introduction to applications of nonlinear programming: optimal control problems, structural design, mechanical design, electrical networks, water resources management, stochastic resource allocation, location of facilities, financial engineering problems.

Review of convex functions and convex optimization.

Nonlinear programming problems, unconstrained problems, problems with inequality and equality constraints, second-order necessary and sufficient optimality conditions for constrained problems (Fritz John and Karush-Kuhn-Tucker conditions).

Duality and optimality conditions in nonlinear programming.

Algorithms for solving NLPs: The line search methods, method of feasible directions.

Focus on special application in one of the following areas: Financial engineering, supply chain management, airline optimization, production planning.

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