

**School of Mathematics and Statistics  
University of Hyderabad**

**Courses in Outcome Based Education format**

# 1 Foundation Courses

## 1.1 Bridge Mathematics, FN 102/MM 102

**Name of the Academic Program:** Foundation course for any program across the university other than for those offered by School of Mathematics and Statistics

**Course Code:** FN 102/MM 102

**Title of the Course:** Bridge Mathematics

**L-T-P per week:** 3-0-0

**Credits:** 3

**Prerequisite Course/Knowledge (if any):**

10th Standard Mathematics

**Course Learning Outcomes (CLO's) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Prove the binomial theorem and apply it to find the expansions of any  $(x + y)^n$  and also, solve the related problems
- CLO-2: Find the various sequences and series and solve the problems related to them.
- CLO-3: Explain the principle of counting and apply it to find the number of permutations and combinations in different cases.
- CLO-4: Explain various trigonometric ratios and find them for different angles, including sum of the angles, multiple and submultiple angles, etc. Also, they can solve the problems using the transformations.
- CLO-5: Find the limit and derivative of a function at a point, the definite and indefinite integral of a function.
- CLO-6: Find the points of min/max of a function.

Mapping of Course Learning Outcomes (CLOs) with Program Learning Outcomes (PLOs) and Program Specific Outcomes (PSOs)

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	1	1	1	1	1	1	1	1
CLO2	2	1	1	2	2	1	2	1
CLO3	2	1	1	2	2	1	2	1
CLO4	1	1	1	1	1	1	2	1
CLO5	1	1	1	1	1	1	2	1
CLO6	2	1	1	1	1	1	2	2

### Syllabus:

- **Unit I:** Algebra: Binomial theorem, General term, middle term, problems based on these concepts, sequences and series (Progressions). Fundamental principle of counting. Factorial n. Permutations and combinations, Derivation of formulae and their connections, simple applications, combinations with repetitions, arrangements within groups, formation of groups.
- **Unit II:** Trigonometry: Introduction to trigonometric ratios, proof of  $\sin(A + B)$ ,  $\cos(A + B)$ ,  $\tan(A + B)$  formulae, multiple and sub multiple angles,  $\sin(2A)$ ,  $\cos(2A)$ ,  $\tan(2A)$  etc., transformations - sum into product and product into sum formulae, inverse trigonometric functions, sine rule and cosine rule.
- **Unit III:** Calculus: Limits, standard formulae and problems, differentiation, first principle,  $uv$  rule,  $u/v$  rule, methods of differentiation, application of derivatives, integration - product rule and substitution method.

### References / Reading Material :

1. NCERT class XI and XII text books.
2. Any State Board Mathematics text books of class XI and XII.

## 1.2 Basic Algebra, FN 135

**Name of the Academic Program:** Foundation course MSc in Statistics and IMSc in Mathematical Sciences

**Course Code:** FN 135

**Title of the Course:** Basic Algebra

**L-T-P per week:** 3-0-0

**Credits:** 3

### Course description and objective

This course covers the basic concepts of discrete mathematics used in Statistics that allows students to develop the ability to reason formally. In this course, we will focus on the following topics such as sets, functions, counting principles, groups and its applications.

### Prerequisite Course/Knowledge (if any):

This course is a foundation course for Statistics students, and it doesn't require any prerequisites.

### Course Learning Outcomes (CLO's) (5 to 8)

After completion of this course successfully, the students will be able to

CLO-1: Use mathematically correct terminology and notation.

CLO-2: Construct correct direct and indirect proofs.

CLO-3: Use problem-solving strategy like divide and conquer.

CLO-4: Use counterexamples.

CLO-5: Apply logical reasoning to solve a variety of problems.

CLO-6: Apply combinatorial reasoning, appropriate to the discipline.

Mapping of CLOs with PLOs and PSOs:

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	2	1		1		1		3	1	1	1		1
CLO-2	2	2					1	3	1	1	1		1
CLO-3	2	2					2	1	1	1	1		1
CLO-4	2	2					2	1	1	1	1		1
CLO-5	2	2		3	3		2	2	1	1	1		1
CLO-6	2	2					2	2	1	1	1		1

## Syllabus:

- Unit I (Set Theory):** Finite sets, infinite sets; countability, uncountability; relations and functions; mathematical induction; partial order relations, posets, chains, anti-chains, Zorn's lemma; lub and glb in a poset.
- Unit II (Counting Principles):** Pigeon Hole principle; mathematical induction; principle of inclusion and exclusion.
- Unit III (Introduction to Groups):** Semi-groups, groups, subgroups, generators and evaluation of powers, cosets, Lagrange's theorem.
- Unit IV (More Group theory):** Permutation groups; normal sub-groups, quotient groups; group homomorphisms, automorphisms, isomorphisms; fundamental theorems of group homomorphisms; Cayley's theorem; group actions, Burnside's theorem.

## References / Reading Material :

1. Gallian, J. A. *Contemporary Abstract Algebra*, 9<sup>th</sup>, ed, Cengage learning, 2015.
2. Graham, R. I; Knuth D. E; Patashnik O, *Concrete mathematics*, second edition, Pearson, 2007.
3. Halmos, P. R., *Naive Set Theory*, Springer, 1960.
4. Herstein, I.N. *Topics in Algebra*, 3rd ed. Wiley, New York, 1996.
5. Joshi, K. D., *Foundations of discrete mathematics*, New age international, 2002.

### 1.3 Elements of Probability and Statistics, FN 134

**Course Code:** (MA 304/MM 404): Foundation Course for M.Sc in Mathematics/Applied Mathematics and I.M.Sc in Mathematical Sciences

**Title of the Course:** Elements of Probability and Statistics

**L-T-P per week:** 3-0-0

**Credits:** 3

**Course Overview:** This course introduces the notion of Uncertainty and Randomness, Probability & Random variables and Basic Data Analysis. To this end the students learn about Random experiments, their Sample Spaces and Events as elements of a Sigma Algebra. Students in the course will learn about existence of Moments. Several Discrete and Continuous random variables will be discussed and their limiting distributions studied.

#### **Course Learning Outcomes (CLO's)**

After completion of this course successfully, the students will be able to

- CLO-1: Identify uncertainty in Nature, other phenomena and processes.
- CLO-2: Recognize different random variables in Nature and other phenomena and identify parameters.
- CLO-3: Write the sample space for any random experiment.
- CLO-4: Determine probabilities of events and probability distributions of random variables.
- CLO-5: Determine whether or not moments exist of any given random variable and if so, to determine them.
- CLO-6: Use tools like Probability Generating function and Moment generating functions to study distributions of some functions of random variables.
- CLO-7 Determine what all data can tell about the data and when anything 'more' can be said, based on the data.

## Syllabus

- Unit I:** Random Experiments: Random Experiments, Sample Spaces, Sets, Events, Algebras. Elements of Counting-Permutations, Combinations.
- Unit II:** Probability Measures: Axiomatic definition of Probability Measure - examples including the classical definition for finite sample spaces. Properties of Probability measure- Theorem of total probability. Conditional Probability, Bayes' Theorem. Independence of events, Conditional independence.
- Unit III:** Random Variables: Definition of a real valued random variable, Distribution Function of a random variable - its properties. Discrete and Continuous random variables. Existence of Moments, Probability and Moment Generating Functions of random Variables. Chebychev's inequality,
- Unit IV:** Some commonly discussed Discrete and Continuous random variables: Discrete random variables: Bernoulli, Binomial, Poisson, Geometric, Negative Binomial, Hypergeometric random variables, Continuous random variables: Uniform, Normal, Exponential, Gamma, Beta random variables-their distributions, Moments, Probability generating and Moment Generating Functions.
- Unit V:** Independence of random variables. Limiting distributions of some random variables, distributions of functions of random variables, Joint distributions of discrete random variables, marginal distributions, conditional distributions, conditional expectation, variance.
- Unit VI:** Data Work: Different kinds of variables, data display of such variables. Frequency curves, Empirical measures of Location, Spread. Data on two or more variables: Scatter Plots, Correlation Coefficient. Fitting of distributions.

### References / Reading Material :

1. Feller, W., Introduction to Probability Theory and its Applications, third edition, Wiley Eastern, 1978.
2. Ross, S., A First Course in Probability, sixth edition, Pearson Education, 2007.
3. Prakasa Rao, B. L. S., A First Course in Probability and Statistics, World Scientific, 2009.
4. Moore, David.S , Notz, W.I., Flinger, M.A.: The Basic Practice of Statistics, WH Freeman and Company, 1994.

## 1.4 Statistics for All, FN 201

**Course Code:** (FN-201): Foundation course for all students of UoH

**Title of the Course:** Statistics for All

**L-T-P per week:** 3-0-0

**Credits:** 3

**Course Overview:** The main purpose is to convey ideas of Statistics and Probability to students of this university other than those from the School of Mathematics and Statistics. It is specially designed for students who have not had Statistics and/or Mathematics training after school. We expose to students ideas of Statistics and probability without any Mathematics or proofs or derivations.

**Pre-requisite Course/Knowledge (if any):** Basic Mathematics at High School Level

### **Course Learning Outcomes (CLO's)**

After completion of this course successfully, the students will be able to

- CLO-1: Identify different kinds of variables - Binary, Ordinal, Quantitative and display data in figures and tables effectively and efficiently.
- CLO-2: Compute the various measures of central tendency and spread and demonstrate clarity on their strengths and weaknesses.
- CLO-3: Compute probabilities of events, apply Bayes' theorem and compute expected values and variances of some commonly discussed random variables.
- CLO-4: Explain what is meant by a variable being Normally distributed and test for same for any given data. Histogram,  $q - q$  plots and explain what estimators and estimates are.
- CLO-5: Explain the importance of Statistical Inference based Sampling studies and Designed experiments rather than just draw conclusions based on one's observations.

### **Syllabus**

**Unit I:** Working with data: Populations, units, variables, Pictorial, graphical representation of data and their distributions, bar charts, pie-charts, histograms and interpreting them. Tabular representation



of data, Numerical measurements from data - measures of central tendencies, variations, 5 point summaries, box-plots and interpreting them. Distribution of data, the Normal distribution. Using the Normal Distribution table. When data are on two variables scatter plots, measures of association especially correlation, two way tables, joint, marginal distributions.

**Unit II:** Basic Probability: Random experiments, sample spaces, events, probability, conditional probability and independence, Bayes Theorem and its uses. Random Variables, Bernoulli, Binomial, Poisson and their distributions, some continuous random variables especially the Normal random variable and its features.

**Unit III:** Data collection - Sampling: Sampling-what is a good sample and a bad sample, from data to population, learning about it from the data so obtained. Simple random sampling, Stratified random sampling. Principles of Experimental Designs - why and how they are done. Examples of a bad design and good design for a problem.

**Unit IV:** Estimation: Parameters and Statistics, Estimators, desirable qualities of a good estimator- especially unbiasedness, low mean square error, Consistency. Estimation from samples. Method of maximum likelihood and moments. Estimation of proportions of attributes. Hypothesis Testing: The problem of Statistical Hypothesis testing, Null and alternate hypotheses, how they are stated, type-1,type-2 errors. Some problems of hypothesis testing for a single normal distribution and two normal distributions. Tests of hypothesis on proportions. p- value. Confidence intervals.

### References / Reading Material

1. Moore,David.S , Notz, W.I., Flinger,M.A. The Basic Practice of Statistics, WH Freeman and Company, 1994.
2. Moore,David.S, Mc Cabe,G.P, Craig,B.A. Introduction to the practice of Statistics, WHFreeman and Company, New York, 2009.
3. Tukey, J.W. Exploratory Data Analysis, Pearson, 1977.

## 2 Core and Elective Courses

### 2.1 Math-I (Linear Algebra), MM 103

Name of the Academic Program: I.M.Sc. in Mathematics

Course Code: MM 103

Title of the Course: Math-I(Linear Algebra)

L-T-P per week: 3-0-0

Credits: 3

Prerequisite Course/Knowledge (if any): Nil

#### Course Learning Outcomes (CLO's) (5 to 8)

After completion of this course successfully, the students will be able to

CLO-1: Explain basics of matrices including definitions.

CLO-2: Solve numerical problems based on basics of matrices

CLO-3: Solve systems of linear equations using row reduced echelon matrices.

CLO-4: Define complex numbers, polar coordinates and triangle inequality

CLO-5: Apply principles of 2 dimensional geometry to solve simple problems

CLO-6: Apply principles of 3 dimensional geometry to solve simple problems

Mapping of CLOs with PLOs and PSOs.

	PLO						PSO	
	1	2	3	4	5	6	1	2
CLO1	3						1	1
CLO2	3						2	2
CLO3	3						2	2
CLO4	3						1	1
CLO5	3						3	3

## Syllabus:

- **Unit I:** Basic concepts of matrices, multiplication of matrices by scalars, addition and multiplication of matrices, transpose, trace and determinant of a matrix, rank and inverse of a matrix, special matrices such as Hermitian, Unitary matrices.
- **Unit II:** System of linear equations, solution by Cramer's rule, row reduced Echelon matrices, existence and general properties of solutions, eigenvalues, eigenvectors, diagonalisation of matrices, functions of matrices and Cayley- Hamilton theorem.
- **Unit III:** Algebra of complex numbers, polar form, Argand diagram, triangle inequality. Addition of vectors, dot product, cross product and their geometric interpretation, triple product, area and volume in terms of vector products.
- **Unit IV:** Coordinate system, distance formula, section formula, area of triangle, straight lines, angle between two lines, concurrent lines, distance between two lines, conic sections.
- **Unit V:** Cartesian coordinates in 3-D, distance between points, direction cosines, direction ratios and their properties, equation of a plane using given data, equation of a straight line in different forms, image of a point with respect to a plane, distance between a point and a plane along a straight line, equation of a circle, sphere.

## References / Reading Material :

1. Linear Algebra, Kenneth Hoffman and Ray Kunze, Pearson, 1971.
2. Linear Algebra : A geometric approach, S. Kumaresan, Prentice Hall of India, 2004.
3. Calculus and Analytic Geometry, George Thomas and Ross Finney, Addition Wesley, 1965.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	10	Units-I, II	Students are able to find the rank of the given matrix. They can compute the inverse of the square matrices (of small sizes) of full rank. They can solve a system of linear equations. They can find the eigenvalues, eigenvectors of given square matrix. Moreover students can diagonalize the given diagonalizable matrix of small size. Students can discuss the solvability of a given system of linear equations. They are able to solve the system whenever it admits a solution.
3.	10	Unit-III	Students are able to apply the properties of the cross product and dot products to solve problems.
4.	12	Unit-IV	The students are able to apply the formulae that they learn to solve problems in the 2-D co-ordinate geometry.
5.	13	Unit-V	Students are able to apply the formulae that they learn to solve problems in the 3-D co-ordinate geometry.

## 2.2 Math-II (Multivariable Calculus), MM 152

**Name of the Academic Program:** I.M.Sc. in Mathematics

**Course Code:** MM 152

**Title of the Course:** Math-II (Multi variable Calculus)

**L-T-P per week:** 3-0-0

**Credits:** 3

**Prerequisite Course/Knowledge (if any):**

Students should know how to calculate the derivative and the integration of real-valued continuous functions over subsets of real-line.

**Course Learning Outcomes (CLO's) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Calculate integration of functions by substitution, by partial fractions and by parts.
- CLO-2: Apply basic properties of definite integrals to calculate definite integrals.
- CLO-3: Define spherical and cylindrical co-ordinate systems.
- CLO-4: Calculate the double integral over a rectangle and a region.
- CLO-5: Calculate triple integrals, applying change of order of integration.
- CLO-6: Define change of variables and Jacobian.
- CLO-7: Parameterize curves and surfaces to find line integrals and surface integrals respectively.
- CLO-8: Apply Green's theorem to find line integrals, Stokes' theorem to find surface integrals and Gauss' theorem to find volume integral.

Mapping of CLOs with PLOs and PSOs.

	PLO						PSO	
	1	2	3	4	5	6	1	2
CLO1	1		1	1		1	1	
CLO2	1							
CLO3	1					1		
CLO4	1	1		1				
CLO5	2	1		1	1		1	
CLO6	1		1					
CLO7	2	1	1	1	1	1	1	
CLO8	2	1	2	1	2	1	1	

### Syllabus:

- **Unit I: (Integration):** To resolve a proper fraction  $P(x)/Q(x)$  into its simplest set of partial fractions. Integration of functions by substitution, by partial fractions, by parts etc. Definite integrals, basic properties of definite integrals and evaluation of definite integrals.
- **Unit II:(Double and triple integrals ):** Brief introduction to co-ordinate systems - spherical and cylindrical systems. Double integral over a rectangle, double integral over a region, change of order of integration. Triple integral.
- **Unit III:(Differential operators):** Change of variables and Jacobian. Vector fields, gradient, divergence, curl, vector calculus identities.
- **Unit IV:(Line integrals):** parametric curves, line integrals, path dependence, fundamental theorems of line integrals, conservative fields.
- **Unit V:(Theorems of Green, Gauss and Stokes):** Application of Greens theorem in 2-D, parametric surfaces, surface of revolution, surface integrals, applications of Stokes theorem and Gauss divergence theorem, Green's identities.

### References / Reading Material :

1. Apostol, Tom M., *Calculus. Vol. II: Multi-variable Calculus and Linear Algebra, with Applications to Differential Equations and Probability*, second edition, Blaisdell Publishing Co. Ginn and Co., Waltham, Mass.-Toronto, Ont.-London, **1969**.

2. Grewal, B. S., *Higher Engineering Mathematics*, Khanna Publications, **2001**.
3. Thomas George and Finney Ross, *Calculus and Analytic Geometry*, Addition Wesley, **1995**.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	10	Unit-I	Students are able to compute the definite and indefinite integrals using various techniques discussed in the course.
2.	20	Units-II,III	Students are able to compute multiple integrals.
3.	20	Units-IV, V	Students are able to compute line, surface and volume integrals.

## 2.3 Math-III A (Ordinary Differential Equations), MM 202

**Name of the Academic Program:** I.M.Sc. in Mathematics

**Course Code:** MM 202

**Title of the Course:** Math-III A (Ordinary Differential Equations)

**L-T-P per week:** 3-0-0

**Credits:** 3

**Prerequisite Course/Knowledge (if any):** Calculus

### Course Learning Outcomes (CLO's) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1: Explain the genesis of ordinary differential equations.
- CLO-2: Apply various techniques of getting exact solutions of solvable first order differential equations and linear differential equations of higher order.
- CLO-3: Apply Picard's method of obtaining successive approximations of solutions of first order differential equations, passing through a given point in the plane and Power series method for higher order linear equations, especially in cases when there is no method available to solve such equations.
- CLO-4: Explain the concept of a general solution of a linear differential equation of an arbitrary order and apply a few methods to obtain the general solution of such equations.
- CLO-5: Explain the Laplace transforms and Fourier series and their applications.

Mapping of CLOs with PLOs and PSOs.

	PLO						PSO	
	1	2	3	4	5	6	1	2
CLO1	2						3	3
CLO2		3					3	3
CLO3		3					3	3
CLO4				3		2	3	3
CLO5	3						3	3

## Syllabus:

- **Unit I:** First order differential equations : Order and degree of a differential equation, first order equations: variables separable method, homogeneous equations of degree zero, non-homogeneous equations, exact equations, integrating factor, linear equations, Bernoulli's equation.
- **Unit II:** First order higher degree equations : solvable for  $x$ ,  $y$  and  $p$ . Clairaut's form and singular solutions. Picard's method of successive approximations and the statement of Picard's theorem for the existence and uniqueness of the solutions of the first order differential equations.
- **Unit III:** Higher order homogeneous linear equations with constant coefficients, second order homogeneous linear equation with variable coefficients, variation of parameters,  $2 \times 2$  autonomous system of equations, power series solution, meaning of existence and uniqueness of a solution and some counter examples.
- **Unit IV:** Laplace Transform : Definition, L.T. of some elementary functions, effect of L.T. on translation, scaling, convolution. Inverse Laplace transform, applications of L.T. to ODE.
- **Unit V:** Fourier series : Fourier series of a periodic function, half range Fourier series.
- **Unit VI:** Sets, relations and functions : Sets, relations, equivalence, partial ordered relations, mathematical induction, elements of mathematical logic.

## References / Reading Material :

1. Advanced Engineering Mathematics, Erwin Kreyszig, Wiley and Sons, 2011.
2. Differential equations with applications and historical notes, George F. Simmons, McGraw Hill Inc, 1972.
3. Elementary Differential Equations, William E. Boyce (Author), Richard C. DiPrima, Wiley and Sons, 2012.
4. Elementary Differential Equations, William E. Boyce (Author), Richard C. DiPrima, Wiley and Sons, 2012.
5. Daniel A. Murray (2003). Introductory Course in Differential Equations, Orient.



## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	5	Unit-I	Students are able to solve first order ODEs using various methods discussed in the course.
2.	5	Unit-II	Students are able to find solutions of first order nonlinear ODEs with special structure. They can compute Picard sequences of approximations corresponding to the initial value problems.
3.	10	Unit-III	Students are able to solve second order linear ODEs and system of ODEs with constant coefficients using various methods discussed in the course.
4.	10	Unit-IV	Students are able to compute the Laplace transforms and the inverse Laplace transforms of functions.
5.	5	Unit-V	Students are able to express functions as a trigonometric series.
6.	5	Unit-VI	Students are able to solve problems involving different types of relations and use the principle of mathematical induction to prove mathematical statements.

## 2.4 Math-IIIB (Introductory Probability and Statistics), MM 203

**Course Code:** (MM 203): IMSc in Mathematical Sciences

**Title of the Course:** Math-IIIB (Introductory Probability and Statistics)

**L-T-P per week:** 3-0-0

**Credits:** 3

**Course Overview:** This course introduces the notion of Uncertainty and Randomness, Probability & Random variables and Basic Data Analysis. Students in this course will learn several univariate discrete and continuous random variables and its properties. This course also introduces the ideas of Statistical Inference and its importance in real world applications.

**Pre-requisite Course/Knowledge (if any):** Basic Mathematics at High School Level

### **Course Learning Outcomes (CLO's)**

After completion of this course successfully, the students will be able to

- CLO-1: Identify uncertainty in Nature, other phenomena and processes.
- CLO-2: Recognize different random variables in Nature and other phenomena and identify parameters.
- CLO-3: Write the sample space for any random experiment.
- CLO-4: Determine probabilities of events and probability distributions of random variables.
- CLO-5: Determine moments of widely known distributions.
- CLO-6: Determine what all data can tell about the data and when anything 'more' can be said, based on the data.
- CLO-7 Identify the concept of statistical inference: parameter, statistic, estimator, estimate and pivotal quantity.

## Syllabus

- Unit I:** Random experiments, sample spaces, events, probability measure on events definition, properties, examples. Conditional probability-definition, properties, examples, Bayes theorem, independent events.
- Unit II:** Definition of random variables, standard discrete and continuous random variables -viz. Bernoulli, Binomial, Geometric, Poisson, Exponential, Gamma, Normal. Expectation, variance, other properties.
- Unit III:** Definition of bivariate random variables, joint distributions, covariance and correlation between two random variables, independence, distributions of sums.
- Unit IV:** Data collection methods, types of data, graphical summaries of data, numerical summaries of univariate data, bivariate summaries, measures of association.
- Unit V:** Introduction to statistical inference, population parameters, variable(s) of interest, statistic, estimators as random variables.

### References / Reading Material :

1. Ross, S., A First Course in Probability, sixth edition, Pearson Education, 2007.
2. Ramachandran, K.M. and Tsokos, C.P., Mathematical Statistics with Applications, Academic Press, 2009.
3. Daniels, W.W., Biostatistics: a foundation for analysis in the health sciences, 9th edition, John Wiley & Sons, 2008.
4. Moore, D.S., The Basic Practice of Statistics, W.H. Freeman, 2003.

## 2.5 Math-IVA (Analysis), MM 253

**Name of the Academic Program:** I.M.Sc. in Mathematics

**Course Code:** MM 253

**Title of the Course:** Math-IVA (Analysis)

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):**

First year Maths courses at CIS

**Course Learning Outcomes (CLO's) (5 to 8)**

After completion of this course successfully, the students will be able to

CLO-1: Explain basics of sequences, various types of them and learn epsilon-delta language and prove some basic theorems

CLO-2: Explain the concept of continuous functions with examples and some basic theorems.

CLO-3: Explain differentiability of a function at a point, several examples and basic theorems

CLO-4: Explain integrable functions with examples and some theorems and statement of fundamental theorem of calculus

CLO-5: Apply sequences and integration to study infinite series rigorously

Mapping of CLOs with PLOs and PSOs.

	PLO						PSO	
	1	2	3	4	5	6	1	2
CLO1	3	3		3	3	3	2	2
CLO2	3	3		3	3	3	2	2
CLO3	3	3		3	3	3	3	3
CLO4	3	3		3	3	3	3	3
CLO5	3	3		3	3	3	3	3

## Syllabus:

- **Unit I:** Rational numbers, real numbers, sequences, subsequences, monotonicity, boundedness, convergence, limit of a sequence, Cauchy criterion, Bolzano- Weierstrass theorem.
- **Unit II:** Limit of a function, Continuity - both sequential and epsilon-delta definitions, examples of continuous functions, intermediate value theorem.
- **Unit III:** Differentiation, chain rule, mean value theorems and applications, Taylor's theorem, L'Hospital's rule, maxima-minima problems.
- **Unit-IV:** Integration, Riemann's original definition, statement of fundamental theorem of calculus.
- **Unit V:** Rigorous definition of convergence of infinite series, application of integration to summability of series.

## References / Reading Material :

1. R.G. Bartle and D.R. Sherbert, Introduction to Real Analysis, Wiley and Sons, 2011.
2. Ajit Kumar and S. Kumaresan, A Basic Course in Real Analysis, Chapman and Hal, 2014.
3. Richard R. Goldberg, Methods of Real Analysis, Wiley and Sons, 1976.
4. Tom M. Apostol, Mathematical Analysis, Pearson, 1974.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	5	Unit-I	Students will study sequences and apply them to solve problems.
2.	10	Unit-II	Students will study continuous functions and apply them to solve problems.
3.	10	Unit-III	Students will study differentiable functions and apply them to solve problems
4.	5	Unit-IV	Students will study integrable functions and apply them to solve problems.
5.	10	Unit-V	Students will study series and solve problems

## 2.6 Math-IVB (Algebra), MM 254

**Name of the Academic Program:** I.M.Sc. in Mathematics

**Course Code:** MM 254

**Title of the Course:** Math- IVB (Algebra)

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):**

First year Maths courses at CIS

### Course Learning Outcomes (CLO's) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1: Explain the most important concept- relations in all detail, revisit induction and explain a bit of mathematical logic
- CLO-2: Explain the concept of a group with several examples , thereby getting into basic concepts of abstract Maths,
- CLO-3: Discuss what kind of maps are useful in the study of groups, the important concept of isomorphism between groups and explain proofs of basic theorems in groups.
- CLO-4: Define vector space and explain the important concept of a basis via spanning sets and linearly independent sets.
- CLO-5: Explain linear transformations between vector spaces, in particular all about finite dimensional vector spaces, connection between matrices and linear maps and solve the related problems

Mapping of CLOs with PLOs and PSOs.

	PLO						PSO	
	1	2	3	4	5	6	1	2
CLO1	3	3	3		3	3	1	1
CLO2	3	3	3		3	3	2	2
CLO3	3	3	3		3	3	2	2
CLO4	3	3	3		3	3	1	1
CLO5	3	3	3		3	3	2	2

## Syllabus:

- **Unit I:** Sets, Relations, equivalence relations, partial order, mathematical induction, elements of mathematical logic.
- **Unit II:** Groups, examples, subgroups, order of an element in a group, cyclic groups, normal subgroups, permutation groups, quotient groups.
- **Unit III:** Group homomorphisms, isomorphisms, fundamental theorem of group homomorphisms and applications.
- **Unit IV:** Vector space, examples, subspaces, spanning set, linear dependence, linear independence, basis, dimension, sum of two subspaces.
- **Unit V:** Linear transformations, isomorphism, finite dimensional vector spaces are isomorphic to  $\mathbb{R}^n$ , rank-nullity theorem, dimension of quotient spaces, matrix of a linear transformation with examples.

## References / Reading Material :

1. Contemporary Algebra, Joseph Gallian, Cengage 2012.
2. Topics in Algebra, I.N.Herstein, Wiley and Sons, 1975.
3. Linear Algebra, Kenneth Hoffman and Ray Kunze, Pearson, 1971.
4. Linear Algebra : A geometric approach, S. Kumaresan, Prentice hall of India, 2004.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	5	Unit-1	Students will learn logic and relations and apply them to solve problems.
2	10	Unit-2	Students will learn basics of groups and apply them to solve problems.
3	10	Unit-3	Students will study maps between groups and apply them to solve problems.
4	10	Unit-4	Students will study basics of Linear algebra and apply them to solve problems.
5	5	Unit-5	Students will study basics of linear maps and apply them to solve problems.



## 2.7 Real Analysis-I, MA 301/ST 301/MM 401/ST 401

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics/Statistics

**Course Code:** MA 301/IM 301/MM 401/ST 401

**Title of the Course:** Real Analysis-I

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):**B.Sc. level Real Analysis.

### **Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Explain elementary metric space theory relevant to the study of Euclidean spaces and solve elementary problems about metric spaces [Level II, Level III].
- CLO-2: Explain the behavior of sequences and series of real numbers as well as functions in terms of their convergence and solve elementary problems about their convergence [Level II, Level III].
- CLO-3: Explain basic techniques in one-variable calculus and apply those to solve elementary problems in calculus [Level II, Level III].
- CLO-4: Demonstrate the results in Real Analysis with a critical mind and use those results and the proof-techniques to analyze and decide, with suitable justification, whether a given statement in Real Analysis is true or false [Level IV, Level V].
- CLO-5: Explain the theory and theoretical structure of Real Analysis as a prerequisite for the future study of subjects such as Topology, Complex Analysis, Measure Theory and Functional Analysis [Level II].

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	3	2	1			3	3	2
CLO2	3	2	1			3	3	2
CLO3	3	2	1			3	3	2
CLO4	3	2	2			3	3	2
CLO5	3					3	3	2

### Syllabus:

- **Unit I:** Real number system and its structure, and preliminaries of metric spaces: Archimedean property; infimum and supremum; inequalities such as Bernoulli inequality and Cauchy-Schwarz inequality; countable and uncountable sets; metric spaces; open balls, bounded sets, and sequences in metric spaces; closure, interior, and boundary of a set; dense subset; distance between a point and a set; distance between two sets; nested interval theorem.
- **Unit II:** More of elementary metric space theory relevant to the study of Euclidean spaces: continuous functions between metric spaces; uniform and Lipschitz continuity; totally bounded sets; compactness, Heine-Borel theorem; continuous functions and compactness; connectedness; intermediate value theorem.
- **Unit III:** Sequences and series of real numbers and their convergence: sequences and subsequences of real numbers; convergence and limit of some standard sequences; sequences related to exponential and log functions; monotone sequences; series of real numbers and their convergence; absolute and unconditional convergence; geometric series; tests for convergence such as comparison test, ratio test, and root test; power series and radius of convergence.
- **Unit IV:** Differentiation of one-variable functions: properties of differentiation; product rule and Chain rule; Darboux theorem; Mean value theorems; test for local extrema; L'Hospital's rule; Taylor's theorem; convex functions and differentiation.
- **Unit V:** Sequences and series of functions and their convergence: sequences of functions; point wise and uniform convergence; uniform convergence on compact sets; point wise boundedness and

uniform boundedness; Dini's theorem; equi-continuous family; Arzela-Ascoli theorem; series of real functions and their convergence; Weierstrass M-test; continuity of power series; power series expansion for the exponential function.

### References:

1. Goldberg, Richard R., Methods of Real Analysis, second edition, John Wiley & Sons, Inc., New York-London-Sydney, 1976.
2. Rudin, Walter, Principles of Mathematical Analysis, third edition, International Series in Pure and Applied Mathematics. McGraw-Hill Book Co., New York-Auckland-Dusseldorf, 1976.
3. Bartle, Robert G., The Elements of Real Analysis, second edition, John Wiley & Sons, New York-London-Sydney, 1976.
4. Ross, Kenneth A., Elementary Analysis. The Theory of Calculus, second edition, in collaboration with Jorge M. Lopez, Undergraduate Texts in Mathematics, Springer, New York, 2013.
5. Apostol, Tom M., Mathematical analysis. Second edition. Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1974.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	10	Unit-I	Use the lub property of real numbers to prove elementary results in analysis.
2.	10	Unit-II	Apply properties of continuous functions to solve problems. Identify whether a given set is compact, connected.
3.	10	Unit-III	Identify whether a given sequence/series is convergent.
4.	10	Unit-IV	Solve problems using the results like mean value theorem, Darboux theorem etc.
5.	10	Unit-V	Identify whether a given sequence/series of functions is uniformly convergent. Check whether a given sequence has a convergent subsequence.

## 2.8 Linear Algebra, MA 302/MM 402

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 302/MM 402

**Title of the Course:** Linear Algebra

**L-T-P per week:** 4-0-0

**Credits:**4

**Prerequisite Course/Knowledge (if any):** Nil

**Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Solve systems of equations using basic matrix theory [Level I, Level II]
- CLO-2: Explain Vector Spaces and Linear Transformations with matrix representations [Level I, Level II]
- CLO-3: Find Eigenvalues and eigenspaces of a linear transformation [Level III]
- CLO-4: Find Diagonal/Triangular form of a Matrix [Level III]
- CLO-5: Analyze a linear transformation by finding Rational and Jordan Canonical forms [Level III, Level IV]
- CLO-6: Apply Gram-Schmidt process of orthonormalizations in Inner Product Spaces and solve related problems [Level II, Level IV]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	3	1	3			2	1	1
CLO2	3		3			3		1
CLO3	3	1	3			3		1
CLO4	3	1	3			3		1
CLO5	3	1	3			3		1

## Syllabus:

- **Unit I:** Basics of Matrix Theory Matrices and elementary row/column operations, Row-reduced Echelon form, System of linear equations (homogeneous and non-homogeneous), Inverse of a matrix, Cramer rule
- **Unit II:** Vector Spaces and Linear Transformations Vector Spaces and Subspaces, Basis and dimension, Linear Transformations and their matrix representations, Rank-Nullity Theorem, Quotient Spaces, Direct Sum, Dual of a Vector Space and Construction of Dual Basis.
- **Unit III:** Inner Product Spaces Orthogonality, Cauchy-Schwarz inequality, Gram-Schmidt orthonormalization process
- **Unit IV:** Canonical Forms of linear transformations Eigenvalues and eigenvectors, Minimal and Characteristic polynomials, Cayley-Hamilton Theorem, Diagonalizable and Triangulable operators, Companion matrix, Primary Decomposition Theorem and Cyclic Decomposition Theorem, The Rational and Jordan Canonical forms
- **Unit V:** Bilinear Forms, Matrix representation of bilinear forms, Orthogonality, classification of bilinear forms

## References

1. K Hoffman and R. Kunze, Linear Algebra (2nd edition ) by Prentice-Hall of India, 2005
2. Vivek Sahai, Vikas Bist, Linear Algebra (Narosa Publishing House).
3. M Artin, Algebra Prentice-Hall of India, 2005
4. A. R. Rao, and P. Bhimashankaram, Linear Algebra, second edition, TRIM series 2005.

## Course Plan

S.No	Topics	Course Learning Outcomes	Sessions
1.	Unit-I	Applying basic results and techniques to solve systems of linear equations (homogeneous & non-homogeneous).	8
2.	Unit-II	Apply basic theory of Vector Spaces to solve problems.	16
3.	Unit-III	Applying of Gram-Schmidt process in problems.	10
4.	Unit-IV	Ability to find eigenspaces associated to linear transformations. Able to find triangular form/diagonal form/rational form/Jordan Canonical form of matrices.	16
5.	Unit-V	Identifying the relation between the bilinear forms and quadratic forms and solve problems involving them.	5

## 2.9 Algebra-I, MA 303/ST 303/MM 403

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M. Sc in Mathematics/Applied Mathematics

**Course Code:** MA 303/IM 303/MM 403

**Title of the Course:** Algebra-I

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** The students must have done at least an algebra course in their Bachelor's degree.

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

CLO-1: Understand a Group. [Knowledge, Understand(Level I, Level II)]

CLO-2: Understand and solve some basic problems. [Knowledge, Understand(Level I, Level III)]

CLO-3: Apply these concepts for further study in Algebra. [Knowledge, Understand(Level I, Level III)]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2	3	2	3	2	3	3	1
CLO2	3	3	2	2	3	3	3	1
CLO3	3	3	2	3	2	3	3	1

### Course syllabus

- **Unit I:** Boolean Algebras: Sets, relations, functions, equivalence relations, equivalence classes, partial order relations, posets, chains, anti-chains, Zorn's lemma, lub and glb in a poset, Lattice, distributive lattice, universal upper/lower bonds, complemented distributive Lattices, Boolean lattices/algebra, Finite Boolean lattices.

- **Unit II:** Groups: Groups and Rings: Introduction, Semi-groups, Groups, Subgroup, Generators and Evaluation of Powers, Cosets and Lagrange's Theorem, Permutation Groups, Normal Subgroups, Quotient groups
- **Unit III:** Group homomorphisms, Automorphisms, Isomorphisms, Fundamental theorems of group homomorphisms, Cayley's Theorem, Group actions, Burnside's Theorem, Sylow's Theorems 1st, 2nd, 3rd and their applications.
- **Unit IV:** Structure theorem for finite Abelian groups, Composition series, Jordan-Holder theorem, Nilpotent and solvable groups.

### References

1. Michael Artin, Algebra, 2nd ed. Pearson, Upper Saddle River, NJ, 2011
2. I.N. Herstein, Topics in Algebra, 3rd ed. Wiley, New York, 1996.
3. Dummit and Foote, Abstract Algebra, 3rd ed. Wiley, New York, 2003.
4. P.R. Halmos, Nave Set Theory, Springer, New York, 1991.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	10	Unit- I	Define Equivalence Relation, Partial Order Relation, Posets and give Examples. Define min and max on posets and Lattice and give examples Define distributive lattice and complemented lattice. Give examples. Define Boolean Lattices give examples. After learning, we are able to understand the complete structure of the finite Boolean lattices.
2.	15	Unit-II	Define semi -group, group, subgroups and rings. Examples Define cosets. Comparing cardinalities of cosets. Prove the Lagrange theorem for finite groups. Define permutation groups, transpositions, odd and even permutation. Give some examples and some problems. Define normal subgroup, quotient group. Give examples. After learning, We are able to understand the difficulties in understanding the structure of groups.
3.	15	Unit-III	Define group homomorphisms, isomorphisms, automorphisms, Cayley's theorem. Fundamental theorem of group homomorphism. After learning these topics, we may be able to identify groups up to isomorphism. Define groups acting on a set. Prove Burnside's theorem. Prove class equation. Applications of group actions to prove Sylow's 1st, 2nd and third theorems. Solve some problems. After learning, we are able to solve some problems related to simple groups.
4.	15	Unit-IV	Prove the structure of finite abelian groups. After proving this theorem, we are able understand the complete structure of finite abelian groups. Define composition series. State and prove Jordan Holders theorem and give some examples. Define Nilpotent and solvable groups. These are topics are at introductory level for them to use in Algebra-III.

## 2.10 Numerical Methods and Programming, MA 305/MM 405

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 305/MM 405

**Title of the Course:** Numerical Methods and Programming

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Nil

### **Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Compute approximate values of zeros of polynomial equations, transcendental equations (Level III).
- CLO-2: Compute the interpolating polynomial for a given data and apply various algorithms to find the value of the interpolating polynomial at given a given point (Level III).
- CLO-3: Compute an approximate solution of a system of linear equations and to compute approximate eigenvalues and approximate eigenvectors of a square matrix (Level III).
- CLO-4: Compute an approximate values of definite integrals by applying various forms of Newton-Cotes formulae (Level III).
- CLO-5: Write programs for the algorithms that they learn in Units III to VI in both *C* language as well as MATLAB (Level III).
- CLO-6: Draw 2-D and 3-D plots using MATLAB to get motivation for various theorems, counter examples in Analysis. This can include courses in 2, 3 and 4th semesters also. For example, draw graphs of  $x \sin(1/x)$ ,  $x^2 \sin(1/x)$  to conclude which one has more total variation (Level III).



Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2	2	3	1	3	3	2	1
CLO2	2	2	3	1	3	3	2	1
CLO3	2	2	3	1	3	3	2	1
CLO4	2	2	3	1	3	3	2	1
CLO5	2	2	3	1	3	3	2	1
CLO6	2	2	3	1	3	3	2	1

### Course syllabus

- **Unit I:** Flow charts and algorithms, sample C-Programmes: compilation and execution of the programmes. C-alphabet: ASCII Character set, basic data types, variables and constants in C. Operators in C: Hierarchy and associativity. Flow control instructions in C: Decision control (if- else), loop control (for, while, do-while), case control (switch). The break and continue statements, functions, arrays, structures (user defined data types).
- **Unit II:** (At least 10 hands on sessions): MATLAB/Octave, implementation of algorithms which are in Units III - VI.
- **Unit III:** (NUMERICAL COMPUTATION): Representation of integers and fractions, fixed point and floating point arithmetics, error propagation, loss of significance, condition and instability, computational method of error propagation. Root finding: bisection method, secant method, regula-falsi method, Newton-Raphson method.
- **Unit IV:** LU decomposition, Gauss elimination with and without pivoting, Gauss-Jacobi method, Gauss-Seidel method, Power method, Jacobi method to find eigenvalues.
- **Unit V:** Interpolation: Lagrange's interpolation, Newton's divided difference interpolation (forward, backward), Newton-Gregory formulae, Sterling's formula.
- **Unit VI:** Numerical integration: Newton-Cotes (closed type formulae)-trapezoidal rule, Simpson's 1/3 -rd rule, Simpson's 3/8 -th rule.

### References

- 1 Yashavant P. Kanetkar, Let Us C, BPB publications, 13th edition, 2012.

- 2 Yashavant P. Kanetkar, Test Your C Skills, BPB publications, fifth edition, 2009.
- 3 Conte, S. D. and deBoor, C., Elementary Numerical Analysis - An Algorithmic Approach, third edition, McGraw Hill, 1981.
- 4 Henrici, P., Elements of Numerical Analysis, John Wiley & Sons, 1964.
- 5 Froberg, C. E., Numerical Mathematics - Theory and Computer Applications, The Benjamin Cummings Pub. Co. 1985.
- 6 Quarteroni, A.; Saleri, F. and Gervasio, P., Scientific Computing with MATLAB and Octave, third edition, Springer, 2010.
- 7 Rudra Pratap, Getting Started with MATLAB 7: A Quick Introduction for Scientists and Engineers, Oxford University Press, 2005.
- 8 Stoer, J. and Bulirsch, R., Introduction to Numerical Analysis, Texts in Applied Mathematics, Springer, 2002.

### Course Plan

S.No.	Sessions	Topics to cover	Course Learning Outcomes
1.	20	Unit I and II	The students learn to write algorithms and programs in C and Matlab.
2.	7	Unit III	The students learn to solve systems of equations using exact and approximate methods.
3.	7	Unit IV	The students learn numerical techniques to find eigenvalues and eigenvectors in some special cases.
4.	12	Unit V	The students learn various type of interpolation formulae.
5.	9	Unit VI	The student learn various Newton Cotes formulae for numerical integration.

## 2.11 Real Analysis-II, MA 356/MM 451

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 356/MM 451

**Title of the Course:** Real Analysis-II

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** The students who have attended **Real Analysis I (code MM401)** and **Linear Algebra (code MM402)** are eligible for taking this course.

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1: Calculate existence or nonexistence of limits, derivatives of scalar fields, and vector fields [Level III].
- CLO-2: Calculate derivatives using the chain rule [Level III].
- CLO-3: Optimize non-linear functions using the Lagrange multipliers to compute multiple integrals using the Fubini's theorem, and by the change of variables [Level III].
- CLO-4: Parametrize curves and surfaces to find line integrals, surface integrals [Level III].
- CLO-5: Apply the Green's, Gauss', and Stokes' theorems to find line, surface and volume integrals [Level III].

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1		2		2	2	2	2	
CLO2	2	2	2			2	2	
CLO3	2	2	2	2	2	2	2	
CLO4	2	2	2	2	2	2	2	
CLO5	2		2	2		2	2	

## Course syllabus

- **Unit I: (Functions of several variables):** Functions of several variables, directional derivative, partial derivative, total derivative, Jacobian, chain rule and mean value theorems, higher derivatives, interchange of the order of differentiation, Taylor's theorem
- **Unit II: (Inverse and implicit mapping theorem):** Inverse mapping theorem, implicit function theorem,
- **Unit III: (Extremum problems):** Extremum problems, extremum problems with constraints, Lagrange's multipliers method.
- **Unit IV: (Multiple integrals):** Multiple integrals, properties of integrals, existence of integrals, iterated integrals, change of variables.
- **Unit V:(Theorems of Green, Gauss and Stokes):** Curl, gradient, div, Laplacian in cylindrical and spherical coordinates, line integrals, surface integrals, theorems of Green, Gauss and Stokes.

## References

1. Apostol, Tom M., Mathematical Analysis, second edition, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1974. (Chapter 6,7,10 and 11.2).
2. Apostol, Tom M., Calculus. Vol. II: Multi-variable Calculus and Linear Algebra, with Applications to Differential Equations and Probability, second edition, Blaisdell Publishing Co. Ginn and Co., Waltham, Mass.-Toronto, Ont.-London, 1969.
3. Munkres, James R., Analysis on Manifolds, Addison-Wesley Publishing Company, Advanced Book Program, Redwood City, CA, 1991.
4. Spiegel, Murray R., Schaum's Outline of Vector Analysis, Schaum's Outline Series, 1959.
5. Spivak, Michael, Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus, W. A. Benjamin, Inc., New York-Amsterdam, 1965.
6. Moskowitz, M., and Paliogiannis F., Functions of Several Real Variables, World Scientific, 2011.

## Course Plan

S.No.	Sessions	Topics to cover	Course Learning Outcome
1.	13	Unit I	Students learn to check the existence of limits, derivatives of functions and compute them whenever they exist.
2.	7	Unit II	Students are able to apply inverse function theorem and implicit function theorem to solve problems not only in this course but also in other courses.
2.	5	Unit III	Students learn to apply Lagrange multiplier method to find extremizers under constraints.
3.	7	Unit IV	Students can analyze the additional hypothesis to define the Riemann integrals higher dimensions. Students can evaluate integrals using various techniques like change of variables, Fubini's theorem etc.
4.	12	Unit V	Students can compute line, surface, volume integrals and explain the relation between them via Green's, Stokes, Gauss' theorems.

## 2.12 Measure and Integration, MA 358/MM 452

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 358/MM 452

**Title of the Course:** Measure and Integration

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Real Analysis and Linear Algebra.

### **Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Interpret the theory of outer measures, sigma-algebras, and measure spaces, and solve elementary problems about them [Level II, Level III].
- CLO-2: Interpret the properties of Lebesgue measure, construction of measures, measurable functions, and solve elementary problems about them [Level II, Level III].
- CLO-3: Interpret integrable functions on measure spaces,  $L_p$  spaces, and product measures, and solve elementary problems about them [Level II, Level III].
- CLO-4: Analyze the proofs of results in Measure Theory with a critical mind and use those results and the proof-techniques to analyze and decide, with suitable justification, whether a given statement in Measure Theory is true or false [Level IV, Level V].
- CLO-5: Interpret the theory and theoretical structure of Measure Theory as a prerequisite for the future study subjects such as Ergodic Theory, Fourier Theory, and Operator Theory [Level II].

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2			1		3	3	2
CLO2	2			1		3	3	2
CLO3	2			1		3	3	2
CLO4	2			2		3	3	2
CLO5						3	3	2

### Syllabus:

- **Unit I:** Riemann-Stieltjes integral and outer measures: a brief review of Riemann-Stieltjes integral; functions of bounded variation; Jordan outer content.
- **Unit II:** Lebesgue outer measure; null sets; sigma-algebras, and measure spaces, Borel sigma-algebra in the Euclidean space.
- **Unit III:** Measures, construction of measures, and Lebesgue measure: measures; completion of a measure space; constructing measures from outer measures; Lebesgue measure and its properties; Steinhaus' theorem and Lebesgue density theorem.
- **Unit IV:** Measurable functions and limit theorems: measurable functions; Borel and Lebesgue measurability; types of convergence for a sequence of measurable functions; Egorov's theorem and Riesz theorem; approximating measurable functions on the Euclidean space with simple functions, step functions, and continuous functions; Lusin's theorem.
- **Unit V:** Monotone convergence theorem; Fatou's lemma; Lebesgue's dominated convergence theorem; Lebesgue's criterion of Riemann integrability; absolutely continuous functions and Fundamental theorem of Calculus. Vitali's set; Vitali's covering lemma, differentiability of monotone functions; Lebesgue's differentiation theorem (proof optional).
- **Unit VI:**  $L^p$  spaces, and product measures:  $L^p$  spaces; Riesz-Fischer theorem at least for  $p = 1$ ; denseness of continuous functions with compact supports in  $L^p$  for  $1 \leq p < \infty$ ; necessity and sufficiency for convergence in  $L^p$ ; Fubini-Tonelli theorem (proof optional).

## References

1. Apostol, Tom M., Mathematical Analysis, second edition, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1974.
2. Bartle, Robert G., The Elements of Real Analysis, second edition, John Wiley & Sons, New York-London-Sydney, 1976.
3. Royden, H. L., Real Analysis, third edition, Macmillan Publishing Company, New York, 1988.
4. de Barra, G., Measure Theory and Integration, New Age International Pvt. Limited, 1981.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	12	Unit-I	Students are able to verify whether a given function is of bounded variation and analyze the analogy, relation between the Riemann integral and Riemann-Stieltjes integrals.
2.	14	Units-II, III	Students are able to apply the elementary results and characterizations of measurable sets to solve problems.
3.	12	Unit-IV	Students are able to apply the properties of measurable functions to solve problems. Analyze the way of defining the integrability of measurable functions (via simple functions, nonnegative functions) and use the same density arguments to prove various results starting from simple functions.
4.	10	Unit-V	Students are able to apply the monotone/dominated convergence theorem in different contexts to interchange the limit and integral.
5.	7	Unit-VI	Students are able to apply the elementary inequalities that $L^p$ functions satisfy to solve problems.



## 2.13 Topology-I, MA 359/MM 453

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 359/MM 453

**Title of the Course:** Topology-I

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Real analysis-I

### **Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Explain the notion of completeness and compactness of metric spaces and solve elementary problems involving these notions and the basic concepts about topological spaces. [Level II, Level III].
- CLO-2: Explain the notion of base and subbase, product topology, separation axioms, countability axioms, and solve elementary problems involving these notions [Level II, Level III].
- CLO-3: Explain the notion of compactness, connectedness, quotient topology, and spaces of continuous functions, and solve elementary problems involving these notions [Level II, Level III].
- CLO-4: Analyze the proofs of results in Topology with a critical mind and use those results and the proof-techniques to analyze and decide, with suitable justification, whether a given statement in Topology is true or false [Level IV, Level V].
- CLO-5: Relate the theory and theoretical structure of Topology as a prerequisite for the future study subjects such as Functional Analysis, Algebraic Topology, and Differential Topology [Level II].

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2			1		3	3	2
CLO2	2			1		3	3	2
CLO3	2			1		3	3	2
CLO4	2			2		3	3	2
CLO5						3	3	2

### Syllabus:

- **Unit I:** Review of metric spaces and definition of topological spaces: a brief review of the theory of metric spaces with emphasis on completeness and compactness; topological spaces; subspace topology.
- **Unit II:** Basic notions about topological spaces-I: closure, interior, and boundary of a set; limit points of a set; isolated points; dense set; nowhere dense set; set of first category; set of second category; Baire category theorem.
- **Unit III:** Basic notions about topological spaces-II: continuous functions between topological spaces; base and subbase; projections and the product topology; separable spaces; first countable spaces; second countable spaces.
- **Unit IV:** Separation axioms, and compactness: Hausdorff spaces; regular spaces; normal spaces; Urysohn's metrization theorem; compact spaces and their basic properties; continuous functions and compactness.
- **Unit V:** Special types of topological spaces: locally compact spaces; connected and path connected spaces; totally disconnected.
- **Unit VI:** Advanced topics in Topology: Alexander subbase theorem; Tychonoff theorem; quotient maps and the quotient topology; the space of continuous real -valued (or complex-valued) functions on a compact space; Tietze extension theorem.

### References

1. Dugundji, James, Topology, Allyn and Bacon Series in Advanced Mathematics, Allyn and Bacon, Inc., Boston, Mass.-London-Sydney, 1978.

2. Munkres, James R, Topology: A First Course, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1975
3. Kelley, John L., General Topology, Graduate Texts in Mathematics, No. 27, Springer-Verlag, New York-Berlin, 1975.
4. Willard, Stephen, General topology, Reprint of the 1970 original [Addison-Wesley, Reading], Dover Publications, Inc., Mineola, NY, 2004.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	20	Unit-I	Understand Complex plane, holomorphic functions, branches of logarithm, Mobius maps, and complex integration along a path, and solve elementary problems about them. Study the proofs of results about Complex plane, holomorphic functions, branches of logarithm, Mobius maps, and complex integration along a path with a critical mind and use those results and the proof-techniques to analyze and decide, with suitable justification, whether a given statement about them is true or false.
2.	17	Unit-II, III	Understand Cauchy's theory, power series expansion, and fundamental theorems about holomorphic functions, and solve elementary problems about them. Study the proofs of results about Cauchy's theory, power series expansion, and fundamental theorems about holomorphic functions with a critical mind and use those results and the proof-techniques to analyze and decide, with suitable justification, whether a given statement about them is true or false.
3.	18	Unit-IV, V, VI	Understand singularities, residues, zeros and poles, automorphism groups, and harmonic functions, and solve elementary problems about them. Study the proofs of results about singularities, residues, zeros and poles, automorphism groups, and harmonic functions with a critical mind and use those results and the proof-techniques to analyze and decide, with suitable justification, whether a given statement about them is true or false.

## 2.14 Algebra-II, MA 360/MM 454

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 360/MM 454

**Title of the Course:** Algebra-II

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** First Semester Linear Algebra and Algebra-I courses

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1 Learn definition and basics of rings, ideals, homomorphisms between rings (Apply)
- CLO-2 Learn important classes of rings like Euclidean Domains, Principal Ideal Domains, Unique Factorization Domains (Apply)
- CLO-3 Learn all about polynomial rings over rings, fields, irreducible polynomials over fields and rings (Apply)
- CLO-4 Learn definition and basics of modules, move on to study modules over special rings (Apply)
- CLO-5 Learn elementary properties of field extensions, algebraic closure, existence of algebraic closure of any field, applications to geometry. (Apply)

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	3			3	3	3	1	2
CLO2	3			3	3	3	2	2
CLO3	3			3	3	3	2	2
CLO4	3			3	3		2	2
CLO5	3			3	3	3	3	3

## Syllabus:

- **Unit I:** Basic concepts in rings, ideals, homomorphism of rings, quotients with several examples.
- **Unit II:** Euclidean domains, principal ideal rings/domains, factorization domains and unique factorization domains.
- **Unit III:** Eisenstein's irreducibility criterion and Gauss's lemma.
- **Unit IV:** Modules, Modules over PIDs, Modules with chain conditions, Hilbert Basis Theorem.
- **Unit V:** Field extensions, algebraically closed fields, finite fields, Ruler and compass techniques etc.

## References

1. Artin, Michael, Algebra, Prentice Hall, Inc., Englewood Cliffs, NJ, 1991.
2. Herstein, I.N. Topics in Algebra, 3rd ed. Wiley, New York, 1996.
3. Dummit, David S. and Foote Richard M., Abstract Algebra, third edition, John Wiley & Sons, Inc., Hoboken, NJ, 2004.
4. Jacobson, Nathan, Basic Algebra, Volume 1, second edition, W. H. Freeman and Company, New York, 1985.
5. Musili, C., Introduction to Rings and Modules, Narosa, 1992.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	11	Unit-I	Students study basics of rings and apply it to solve problems.
2.	8	Unit-II	Students study various kind of domains and apply it to solve problems.
3.	8	Unit-III	Students study polynomial rings and apply it to solve problems.
4.	16	Unit-IV	Students study theory of modules and apply it to solve problems.
5.	12	Unit-IV	Students learn basic field theory and apply it to solve problems.

## 2.15 Ordinary Differential Equations-I, MA 357/MM 455

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 357/MM 455

**Title of the Course:** Ordinary Differential Equations-I

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Calculus and Linear Algebra

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

CLO-1: Solve ODEs in the standard forms [Level III].

CLO-2: Decide whether a given ODE or system of ODEs is well-posed [Level III]

CLO-3: Analyze the asymptotic behavior of solutions to any first order autonomous (system of) ODE(s) without solving it [Level IV].

CLO-4: Construct Green's functions for second order linear differential operators with various boundary conditions [Level III].

CLO-5: Analyze the nature of the critical points using Lyapunov theory [Level IV].

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2		2				3	3
CLO2	3		2			3	3	3
CLO3	3		2			3	3	3
CLO4	2		2			3	3	3
CLO5	3		2			3	3	3

## Course syllabus

- **Unit I:** First order differential equations: Ordinary Differential Equations, mathematical models, first order equations, existence, uniqueness theorems, continuous dependence on initial conditions, Gronwall's inequality and applications.
- **Unit II:** Second order linear differential equations: Wronskian, explicit methods to find solutions, method of variation of parameters; power series solutions: ordinary points, regular singular points, irregular singular points and Frobenius methods; special functions: Legendre and Bessel functions, properties. Oscillation theory, qualitative properties of solutions, Sturm separation and comparison theorems.
- **Unit III:** Two-point boundary value problems: Sturm-Liouville equations, Green's functions, construction of Green's functions, non homogeneous boundary conditions, eigenvalues and eigenfunctions of Sturm-Liouville equations, eigenfunction expansions.
- **Unit IV:** Systems of ordinary differential equations: Existence and uniqueness theorems; homogeneous linear systems, fundamental matrix, exponential of a matrix, non homogeneous linear systems, linear systems with constant coefficients.
- **Unit V:** Nonlinear differential equations: Volterra Prey-Predator model; Phase plane analysis: Autonomous systems, types of critical points, stability for linear systems with constant coefficients, stability of nonlinear systems, method of Lyapunov for nonlinear systems, simple critical points, Poincaré's theorem, limit cycles, statement of Poincaré- Bendixson theorem, examples.

## References

1. Simmons, George F., Differential Equations with Applications and Historical Notes, International Series in Pure and Applied Mathematics, McGraw-Hill Book Co., New York-Dusseldorf-Johannesburg, 1972.
2. Birkhoff, Garrett and Rota, Gian Carlo, Ordinary Differential Equations, fourth edition, John Wiley & Sons, Inc., New York, 1989.
3. Coddington, Earl A. and Levinson, Norman, Theory of Ordinary Differential Equations, McGraw-Hill Book Company, Inc., New York-Toronto-London, 1955.

4. Perko, Lawrence, Differential Equations and Dynamical Systems, Springer, New York, third edition, 2001.
5. Ross, Shepley L., Introduction to Ordinary Differential Equations, fourth edition, John Wiley & Sons, Inc., New York, 1989.
6. Cronin, Jane, Ordinary Differential Equations. Introduction and Qualitative Theory, third edition, Pure and Applied Mathematics, 292. Chapman & Hall/CRC, Boca Raton, FL, 2008.
7. Hirsch, Morris W., Smale, Stephen and Devaney, R. L., Differential Equations, Dynamical Systems and an Introduction to Chaos, Academic Press, 2004.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	9	Unit-I	Students can discuss the well posedness of the given initial value problem. Moreover, they can solve problems using the Gronwall lemma and comparison theorem.
2.	14	Unit-II	Students can solve second order ODEs using the methods like power series method, variation of parameters etc. Moreover, they can apply Sturm separation theorem and comparison theorem to solve problems.
3.	9	Unit-III	Students can construct Green's functions and compute eigenvalues for simple second order linear differential operator with separated/periodic boundary conditions.
4.	8	Unit-IV	Students can compute the exponential of a matrix. They can solve the linear non-homogeneous system of ODEs if a correspondence fundamental matrix is given.
5.	15	Unit-V	Students can analyze the behavior of the critical point of the given $2 \times 2$ system.



## 2.16 Functional Analysis, MA 401/MM 501/AM 501

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 401/MM 501/AM 501

**Title of the Course:** Functional Analysis

**L-T-P per week:** 4-0-0

**Credits:** 4

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1 Explain the notions of completeness, separability, reflexivity, duality of normed spaces and apply them to solve elementary problems (Level-II, Level-III).
- CLO-2 Apply the properties of bounded linear maps between Banach spaces/ Hilbert spaces to solve problems (Level-III).
- CLO-3 Explain the main results of functional analysis like Hahn-Banach theorem, uniform boundedness theorem, open mapping theorem, closed graph theorem and apply them to solve problems (Level-III).
- CLO-4 Apply the special properties that Hilbert Spaces possess (like existence of orthonormal basis, Riesz representation theorem etc) to solve problems (Level-II, Level-III).
- CLO-5 Explain compact operators, spectral theorem for self adjoint compact operators and solve problems involving these concepts. (Level-III)

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	3			3	3	3	2	3
CLO2	3			3	3		2	3
CLO3	3			3	3		2	3
CLO4	3			3	3	3	2	3
CLO5	3			3	3		2	3

## Syllabus:

- **Unit I:** Fundamentals of metric spaces, completion of metric spaces, normed spaces, Holder's inequality, Minkowski's inequality.
- **Unit II:**  $l^p$ -spaces, Banach spaces. Equivalence of norms, Riesz lemma, bounded linear maps, operator norm, separability.
- **Unit III:** Hahn-Banach theorem, dual of normed spaces, reflexivity, open mapping and closed graph theorems, uniform boundedness principle.
- **Unit IV:** Inner product spaces, Hilbert spaces, examples, projection theorem. Bessel's inequality, existence of Schauder basis for separable Hilbert spaces, Riesz representation theorems.
- **Unit V:** Compact operators, finite rank operators, properties of self adjoint compact operators, spectral theorem.

## References:

1. Kreyszig, Erwin, Introductory Functional Analysis with Applications, Wiley Classics Library, John Wiley & Sons, Inc., New York, 1989.
2. Limaye, Balmohan V., Functional Analysis, second edition, New Age International Publishers Limited, New Delhi, 1996.
3. Kesavan, S., Functional Analysis, Trim series, Hindustan Book Agency, 2009.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	11	Unit-I	Students can solve problems involving the topological properties of normed spaces.
2.	11	Unit-II	Students can use the properties of bounded linear maps to solve problems.
3.	11	Unit-III	Students can analyze the proofs of 4 pillars of Functional Analysis and apply them to solve problems.
4.	11	Unit-IV	Students can solve elementary problems involving Hilbert spaces.
5.	11	Unit-V	Students can analyze the properties of compact operators and apply them to solve elementary problems.

## 2.17 Partial Differential Equations-I, MA 402/MM 502/AM 502

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 402/MM 502/AM 502

**Title of the Course:** Partial Differential Equations-I

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Real Analysis-I, Real Analysis -II, Ordinary Differential Equations-I.

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO 1: Apply a suitable technique to solve a given first order/second order partial differential equations based on the type of the equation [Level-III].
- CLO 2: Explain the limitations of the characteristic method that is used to solve first order PDEs and the method of separation of variables which is used to solve second order linear PDEs [Level-II].
- CLO 3: Construct Green's function for the Laplacian with Dirichlet/Neumann boundary conditions when the geometry of the domain is simple.
- CLO 4: Analyze the nature of the solutions to heat equation and the wave equations by observing phenomena like finite speed of propagation and propagation regularizing effect etc.
- CLO 5: Identify various results/tools from the prerequisite courses that are used in the proofs of the theorems given in this course.

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1							3	3
CLO2	2		3				3	3
CLO3	3		2				3	3
CLO4	3		2				3	3
CLO5	3		2				3	3

## Syllabus:

- **Unit I:** First order partial differential equations : Surfaces and curves, classification of first order P.D.E., classification of solutions, Pfaffian differential equations, quasi-linear equations, Lagrange's method, compatible systems, Charpit's method, Jacobi's method, integral surfaces passing through a given curve, method of characteristics for quasi-linear and nonlinear P.D.E., Monge cone, characteristic strip.
- **Unit II:** Second order partial differential equations : Classification of second order semi-linear P.D.E., Hadamard's definition of well-posedness.
- **Unit III:** Wave equation: D'Alembert's solution, vibrations of a finite string, existence and uniqueness of solution, Riemann method.
- **Unit IV:** Laplace's equation: Boundary value problems, maximum and minimum principles, uniqueness and continuity theorems, Dirichlet problem for a circle, Dirichlet problem for a circular annulus, Neumann problem for a circle, theory of Green's function for Laplace's equation.
- **Unit V:** Heat equation: Heat conduction problem for an infinite rod, heat conduction in a finite rod, existence and uniqueness of the solution;
- **Unit VI:** Duhamel's principle for wave and heat equations. Variable separable methods for second order linear partial differential equations. Classification of semi-linear partial differential equations in higher dimensions; Kelvin's inversion theorem; equipotential surfaces.

## References

1. Fritz, John, Partial Differential Equations, second edition, Applied Mathematical Sciences, Vol. 1, Springer-Verlag, Chapter - I, 1978.
2. Weinberger, H. F., A First Course in Partial Differential Equations with Complex Variables and Transform Methods, Wiley, 1965.
3. Sneddon, Ian, Elements of Partial Differential Equations, McGrawHill, NY, 1957; Dover, 2006.
4. Qing, Han, A Basic Course in Partial Differential Equations, ATM, Volume 120 , Indian edition, 2013.

5. McOwen, Robert C., Partial Differential Equations - Methods and Applications, second edition, Pearson India, 2006.
6. Evans, Lawrence C., Partial Differential Equations, AMS-GTM, Vol.19, Indian edition, 2010.
7. Amaranath, T., An Elementary Course in Partial Differential Equations, second edition, Narosa Publishing House, 2012.
- 8 Zauderer, Erich, Partial Differential Equations of Applied Mathematics, third edition, Wiley, 2011.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	15	Unit-I	Students are able to solve first order PDEs using methods due to Lagrange, Charpit, and characteristic method.
2.	5	Unit-II	Student are able to classify second order semilinear PDEs and explain the notion of well posedness.
3.	10	Unit-III	Students are able to solve the wave equation in the domains $\mathbb{R}$ , $[0, \infty)$ and $[0, l]$ . Moreover they can explain the notions of domain of dependence and range of influence.
4.	10	Unit-IV	Students are able to derive solutions of the Poisson equation and the Laplace equations posed in the upper half space, interior/ exterior of balls in $\mathbb{R}^n$ . Moreover students can use the maximum principle to solve problems.
5.	6	Unit-V	Students are able to solve the heat equation using the Fourier transformations and the method of separation of variables. Moreover students can explain the notion of infinite speed of propagation.
6.	8	Unit-VI	Students are able to apply the Duhamel principle to solve problems. Moreover, they are able to classify the semilinear PDE in higher dimensions.

## 2.18 Mathematical Methods, MA 403/MM 503/AM 503

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 403/MM 503/AM 503

**Title of the Course:** Mathematical Methods

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Ordinary Differential Equations-I, Linear algebra, Real analysis-II. Knowledge in Functional Analysis is desirable.

### **Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Compute the Laplace and Fourier transforms of functions and solve linear Volterra integral equations by computing the resolvent kernel or using the Laplace transforms [Level-III].
- CLO-2: Convert linear Volterra/Fredholm integral equations to ordinary differential equations [Level-III].
- CLO-3: Examine the existence of solution of non homogeneous Fredholm integral equations using the Fredholm alternatives [Level-III].
- CLO-4: Write the Euler–Lagrange equations for various types of functionals containing many dependent variables, many independent variables, higher order derivatives etc [Level-II].
- CLO-5: Analyze whether the extremals are minimizers/maximizers of the given functionals [Level-III].
- CLO-6: Find the extremals for moving boundary problems and functionals with constraints [Level-III].

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2	2	2			2	2	
CLO2	2	2	2			2	2	
CLO3	3	3	3			3	3	
CLO4	3	3	3			3	3	
CLO5	3	3	3			3	3	
CLO6	3	3	3			3	3	

### Syllabus:

- **Unit I: Integral transforms**

Laplace transforms: Definitions, properties, Laplace transforms of some elementary functions, convolution theorem, inverse Laplace transformation, applications.

- **Unit II: Fourier transforms**

Definitions, properties, Fourier transforms of some elementary functions, convolution theorems, Fourier transform as a limit of Fourier Series.

- **Unit III: Integral equations**

Volterra integral equations: Basic concepts, relationship between linear differential equations and Volterra integral equations, resolvent kernel of Volterra integral equations, solution of integral equations by resolvent kernel, the method of successive approximations, convolution type equations, solution of integro-differential equations with the aid of Laplace transformation. Fredholm integral equations: Fredholm equations of the second kind, fundamentals, iterated kernels, constructing the resolvent kernel with the aid of iterated kernels, integral equations with degenerate kernels, characteristic numbers and eigenfunctions, solution of homogeneous integral equations with degenerate kernel, non homogeneous symmetric equations, Fredholm alternative.

- **Unit IV: Calculus of variations**

Extrema of functionals: The variation of a functional and its properties, Euler's equation, field of extremals, sufficient and necessary conditions for the extremum of a functional both for weak and strong extrema; Legendre and Weierstrass theorems, Hilbert

invariant integral theorem, conditional extremum, moving boundary problems, discontinuous problems, one sided variations, Ritz method.

**References:**

- 1 Brunt, Bruce van, The Calculus of Variations , Springer-Verlag, New York, 2004.
- 2 Sneddon I. N., The Use of Integral Transforms Tata McGraw Hill, 1972.
- 3 Spiegel, Murray R., Schaum's Outline of Laplace Transforms, Schaum's Outline Series, 1965.
- 4 Gelfand, I. M. and Fomin, S. V., Calculus of Variations, revised English edition translated and edited by Richard A. Silverman, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963.
- 5 Krasnov, M. L.; Makarenko, G. I. and Kiselev, A. I., Problems and Exercises in the Calculus of Variations, translated from the Russian by George Yankovsky, 1975.
- 6 Krasnov, M. L.; Makarenko, G. I. and Kiselev, A. I., Problems and Exercises in Integral Equations, translated from the Russian by George Yankovsky, 1975.
- 7 Kanwal Ram P., Linear Integral Equations, second edition, Birkhauser Boston, Inc., Boston, MA, 1997.
- 8 Pipkin, Allen C., A Course on Integral Equations, Texts in Applied Mathematics, 9, Springer-Verlag, New York, 1991.
- 9 Gibbons, M. M., A Primer on the Calculus of Variations and Optimal Control Theory, Volume-50, AMS, 2009.



## Course Plan

Topics	Hours	Course Learning outcomes
<b>Unit I</b>	10	The students will be able to: 1) compute the Laplace transform of functions using the results like scaling, sheeting, derivative, and integral theorems. 2) compute the inverse Laplace transforms. 3) solve linear initial value problems and boundary value problems by applying the Laplace transforms.
<b>Unit II</b>	4	The students will be able to: 1) compute the Fourier transform of functions. 2) recognize Fourier transform as the limit of Fourier series.
<b>Unit III</b>	18	The students will be able to: 1) convert Volterra integral equations (VIEs) to initial value problems. 2) solve the given VIE by calculating its resolvent. 3) apply the Laplace transforms to solve VIE. 4) convert Fredholm integral equations (FIEs) to boundary value problems. 5) solve the given FIE by calculating its resolvent. 6) compute the eigenvalues and eigenfunctions of Fredholm integral operators with degenerate kernels, and symmetric kernels. 7) analyze the existence and uniqueness of solution of FIE using the Fredholm alternatives.
<b>Unit IV</b>	18	The students will be able to: 1) write the Euler-Lagrange equations to functionals involving first order, higher order, partial derivatives and to functionals with many dependent variables. 2) analyze the existence of non differentiable extremals. 3) compute extremizers of functionals with moving boundary conditions, and constraints. 4) apply the Jacobi necessary condition, sufficient condition and Weierstrass sufficient condition to discuss the existence of minimizers/maximizers.

## 2.19 Complex Analysis-I, MA 404/MM 504/AM 504

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics

**Course Code:** MA 404/MM 504/AM 504

**Title of the Course:** Complex Analysis-I

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Real Analysis and Topology

### **Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1 Identify holomorphic functions, explain branches of logarithm, compute complex integrals, and solve elementary problems [Level II, Level III].
- CLO-2 Explain Cauchy's theory, power series expansion, and fundamental theorems about holomorphic functions, and solve elementary problems about them [Level II, Level III].
- CLO-3 Identify types of singularities, compute residues, explain properties of harmonic functions and solve elementary problems about them [Level II, Level III].
- CLO-4 Analyze the proofs of results in Complex Analysis with a critical mind and use those results and the proof-techniques to analyze and decide, with suitable justification, whether a given statement in Complex Analysis is true or false [Level IV, Level V].
- CLO-5 Interpret the theory and theoretical structure of Complex Analysis as a prerequisite for the future study subjects such as Operator Theory, Riemann Surfaces, Analytic Number Theory, and Complex Algebraic Geometry [Level II].

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2			1		3	3	2
CLO2	2			1		3	3	2
CLO3	2			1		3	3	2
CLO4	2			2		3	3	2
CLO5						3	3	2

### Syllabus:

- **Unit I:** Complex numbers, holomorphic functions, and branches of logarithm: basic properties of complex numbers; holomorphic functions; Cauchy-Riemann equations; complex exponential function; branches of logarithm.
- **Unit II:** Power series, and Mobius maps: complex power series and radius of convergence; Mobius maps on the Riemann sphere, and their properties; biholomorphic maps.
- **Unit III:** Complex integration along a path, Cauchy's integral formula, and power series expansion of holomorphic functions.
- **Unit IV:** Fundamental theorems about holomorphic functions: Liouville's theorem; zeros of holomorphic functions; Maximum modulus principle; Open mapping theorem; Weierstrass' uniform convergence theorem.
- **Unit V:** Winding number, and singularities: winding number; counting zeros in a region bounded by a smooth path; three types of singularities; Casorati-Weierstrass theorem; Laurent series.
- **Unit VI:** A few advanced topics in Complex Analysis: Residue theorem; Argument principle - counting zeros and poles of a meromorphic function; Rouché's theorem; Hurwitz theorem; Schwarz lemma; a brief mention of harmonic functions.

### References

1. Ahlfors, Lars V., Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable, third edition. International Series in Pure and Applied Mathematics, McGraw-Hill Book Co., New York, 1978.

2. Churchill, Ruel V. and Brown, James Ward, Complex Variables and Applications, fourth edition, McGraw-Hill Book Co., New York, 1984.
3. Conway, John B., Functions of One Complex Variable, II, Graduate Texts in Mathematics, 159, Springer-Verlag, New York, 1995.
4. Narasimhan, Raghavan and Nievergelt, Yves, Complex Analysis in One Variable, second edition, Birkhauser Boston, Inc., MA, 2001.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	9	Unit-I	Students are able to solve problems using the properties of holomorphic functions like the maximum principle, C-R equations etc.
2.	9	Unit-II	Students can analyze the properties of power series and apply them to solve problems.
3.	9	Unit-III	Students can apply the Cauchy integral formula to solve problems.
4.	9	Unit-IV	Students can analyze the zeros of holomorphic functions.
5.	9	Unit-V	Students can analyze the nature of singularities and behavior of functions near the singularities.
6.	9	Unit-VI	Students can apply the results like Rouché's theorem, Schwartz lemma etc to solve problems.

## 2.20 Algebra-III, MA 407/MM 505

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc Mathematics

**Course Code:** MA 407/MM 505

**Title of the Course:** Algebra-III

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** This course is compulsory for students who want to pursue Mathematics. Students should have basic knowledge of group theory and field theory.

### **Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1 Define various notions from field extensions, for example algebraic, transcendental, normal separable, the degree of an extension, etc. (Level I)
- CLO-2 Identify and apply the above notions to various examples. (Level II, III)
- CLO-3 State and understand the Galois correspondence. (Level I, II, IV)
- CLO-4 Analyze some examples. (Level V, VI)
- CLO-5 Understand how to apply the Galois correspondence to other areas in mathematics, like classical or algebraic geometry, or in commutative algebra. (Level III, IV)
- CLO-5 Define various concepts from commutative algebra, such as Jordan-Holder series, Noetherian ring, Artinian ring. A student should understand theorems like Nakayama lemma, Chinese remainder theorem. (Level I, II)

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	1			1		3	2	1
CLO2	2	2		2	2	3	2	1
CLO3	3	2		3		3	2	2
CLO4	3	3		3		3	3	2
CLO5	3	2			3	3	3	2
CLO6	2			1			1	1

### Syllabus:

- **Unit I: (Field extensions):** mostly field extension, algebraic extensions, the degree of an extension. Many examples will be discussed for a better understanding of these concepts. Some impossibility theorem in the Ruler and compass constructions proved as an application to algebraic extensions.
- **Unit II:(More field extensions):** The idea behind Galois theory, Normality and separability of extensions, Field degrees, and group orders, normal closures. Discuss many examples and counterexamples for a better understanding of these ideas.
- **Unit III:(The Galois correspondence):** Galois correspondence, explain a close relationship between two different subjects group theory and field extensions along with examples.
- **Unit IV:(Applications of Galois Theory):** Applications of the Galois correspondence to the diverse areas, i.e., solution of equations by radicals, the general polynomial equation, Finite fields, regular polygons, the Fundamental Theorem of Algebra.
- **Unit V:(Transcendental extensions):** Transcendental extensions, Luroth's theorem.
- **Unit VI:(Commutative Algebra):** concepts from commutative algebra: Simple Modules, Jordon-Holder theorem for Modules, Jacobian, and Nilradicals, Nakayama Lemma, Chinese remainder theorem, tensor products. The Artinian ring implies Noetherian.

### References:

1. Michael Artin, Algebra, 2nd ed. Pearson, Upper Saddle River, NJ, 2011

2. Herstein, I.N. Topics in Algebra, 3rd ed. Wiley, New York, 1996.
3. Dummit and Foote, Abstract Algebra, 3rd ed. Wiley, New York, 2003.
4. Lang, Serge, Algebra, revised third edition, Graduate Texts in Mathematics, 211, Springer-Verlag, New York, 2002.
5. Jacobson, Nathan, Basic Algebra, Volume 1, second edition, W. H. Freeman and Company, New York, 1985.
6. Jacobson, Nathan, Basic Algebra, Volume 2, second edition. W. H. Freeman and Company, New York, 1989.
7. Jacobson, Nathan, Lectures in Abstract Algebra, III, Theory of fields and Galois theory, Second corrected printing, Graduate Texts in Mathematics, No. 32. Springer-Verlag, New York-Heidelberg, 1975.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	5	Unit-I	Students are expected to find degrees of finite field extensions, constructing a field extension containing a root of a polynomial over a field. They will also be able to see the impossibility of certain construction by ruler and compass.
2.	10	Unit-II	Students are expected to check whether a given finite extension is Galois or not. They will be able to compute the Galois group of some Galois extensions.
3.	10	Unit-III	Students will be able to understand Galois correspondence, and be able to apply it to some concrete examples.
4.	10	Unit-IV	Students will be able to understand applications of Galois theory to Finite fields, radical extensions, solutions of polynomial equations by radicals, etc.
5.	5	Unit-V	Students will be able to understand the notion of transcendental extensions.
6.	15	Unit-VI	Students will be able to understand a few basic concepts of commutative algebra like Artinian and Noetherian rings, Nakayama lemma. Also, they will be introduced to the concepts of simple modules, Jordan-Holder theorem for modules, etc.

## 2.21 Introduction to Number Theory, MA 406/MM 506

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics

**Course Code:**MA 406/MM 506

**Title of the Course:** Introduction to Number Theory

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Elementary knowledge of Group theory.

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1: Learn modular arithmetic and solving linear Diophantine equations [Level I, Level II, Level III]
- CLO- 2 Understand Fermat's/Euler's theorem and applications to classify integers of special types (sums of two/four Squares) [Level II, Level III]
- CLO- 3 Solve quadratic equations using Quadratic Reciprocity Law [Level I, Level III]
- CLO- 4 Learn Continued fractions and apply to solve some of Diophantine equations (Linear with two variables, Pell's equations etc) [Level II, Level III]
- CLO- 5 Understand Algebraic and Transcendental numbers with explicit examples, prime factorizations in  $\mathbb{Z}[i]$  [Level II]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2		1	1			1	1
CLO2	2		2	2			2	1
CLO3	2		3	3			2	2
CLO4	2		3	3			2	2
CLO5	2		3	3			2	2



## Syllabus:

- **Unit I:** Review of arithmetic in  $\mathbb{Z}$   
Divisibility, gcd & lcm, Bezout's identity, Division Algorithm and Euclidean Algorithm
- **Unit II:** Modular Arithmetic  
Congruence relation, Euler phi-function, Fermat's little theorem, Euler's theorem, Wilson's theorem, CRT, primitive roots and structure of units modulo  $n$ . Solving equations  $X^n = 1$ , Euclid's and Euler's proof of infinitude of primes
- **Unit III:** Quadratic Reciprocity Law  
Quadratic residue, Legendre & Jacobi symbols, Law of Quadratic Reciprocity (at least two proofs) with application in solving quadratic equations, Sums of Two Square theorem and Lagrange Four Square theorem, Primes in arithmetic progressions (*mod*8)
- **Unit IV:** Arithmetic Functions  
Some examples of arithmetic functions, Mobius inversion formula, Big-  $O$  and little-  $o$  notations, Abel's partial summation formula with applications, Prime Number Theorem (statement only) and Dirichlet's theorem on primes in arithmetic progressions (statement only)
- **Unit V:** Continued Fractions  
Existence of and uniqueness of simple continued fraction expansion, periodic continued fraction expansions, Equivalent numbers
- **Unit VI:** Algebraic Numbers  
Algebraic and transcendental numbers, proof of transcendence of  $e$ , rings of algebraic integers (quadratic number fields), Gaussian integers and prime factorizations

## References

1. G H Hardy and E M Wright, *An Introduction to the Theory of Numbers*, 6th edition, Oxford University Press 2008.
2. G A Jones and J M Jones, *Elementary Number Theory*, Springer-Verlag London 1998.
3. K Ireland and M Rosen, *A Classical Introduction to Modern Number Theory*, GTM 84, Springer-Verlag, New York 1990.
4. D M Burton, *Elementary Number Theory*, 7th Edition, McGraw-Hill 2010.

5. T M Apostol, *Introduction to Analytic Number Theory*, Springer-Verlag, New York, Heidelberg 1976.

### Course Plan

S.No.	Sessions	Topics to cover	Course Learning Outcomes
1.	6	Unit-I	Ability to solve linear equations, find GCD
2.	10	Unit-II	Ability to solve $n^{th}$ power residues, Solving Simultaneous linear congruences
3.	16	Unit- III	
4.	8	Unit- IV	Ability to apply Mobius inversion formula and understand the Prime Number Theorem
5.	8	Unit- V	To understand Continued fraction expansions and solutions of Pythagoras and Pell's equations
6.	7	Unit- VI	To learn basics of Algebraic Number Theory

## 2.22 Numerical Analysis, MA 405/AM 505

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Applied Mathematics

**Course Code:** MA 405/AM 505

**Title of the Course:** Numerical Analysis

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Knowledge of Real Analysis, Linear Algebra and Functional analysis, Ordinary Differential Equations and Partial Differential Equations

### **Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1 Apply appropriate algorithms to solve selected problems manually [Level III].
- CLO-2 Compare different algorithms with respect to accuracy and efficiency of solution [Level III].
- CLO-3 Analyze the errors obtained in the numerical solution of problems [Level III].
- CLO-4 Use appropriate numerical methods to determine the solutions to given non-linear equations [Level III].
- CLO-5 Use appropriate numerical methods to determine approximate solutions to systems of linear equations [Level III].
- CLO-6 Use appropriate interpolation techniques and use of Newton Cotes and Gaussian Quadrature to evaluate integrals approximately [Level III].
- CLO-7 Use appropriate numerical methods to determine approximate solutions to ordinary and some partial differential equations [Level III].

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	3	3	2	2	3	3	2	2
CLO2	3	3	2	2	3	3	2	2
CLO3	3	3	2	2	3	3	2	2
CLO4	3	3	2	2	3	3	2	2
CLO5	3	3	2	2	3	3	2	2
CLO6	3	3	2	2	1	3	2	2
CLO7	2	2	2	2	1	3	2	2

### Syllabus:

- **Unit I:** Solution of nonlinear equations: Multi-point iterative methods, fixed point iteration, convergence of methods, polynomial equations, Muller's method, acceleration of convergence.
- **Unit II:** Solution of linear systems: Error and residual of an approximate solution. condition number, theorems of Gershgorin and Brauer, Jacobi method, Power method for Hermitian matrices, inverse power method, convergence of the methods.
- **Unit III:** Polynomial interpolation: Existence and uniqueness of an interpolating polynomial, Hermite interpolation, error of the interpolating polynomials, piecewise-polynomial approximation (up to cubic splines).
- **Unit IV:** Numerical integration: Newton-Cotes Closed and open type formulae, error, composite rules, adaptive quadrature, extrapolation to the limit, Romberg Integration, properties of orthogonal polynomials, Gaussian quadrature. Numerical differentiation.
- **Unit V:** Solution of O.D.E.: Difference equations, Taylor series method, explicit and implicit methods, single and multi-step methods - forward, backward Euler methods, mid-point formula, modified Euler's method and their convergence, Runge-Kutta methods (up to 2nd order O.D.E.), Predictor-Corrector methods, stability of numerical methods, round-off error propagation and control, shooting methods and finite difference methods for B.V.P. (second and fourth order).

- **Unit VI:** Solution of linear P.D.E. (at most second order) : Derivation of difference equations for transport equation, heat equation, wave equation, Laplace equation, Poisson equation, consistency, initial value problems, P.D.E. with Dirichlet and Neumann boundary conditions, stability, Lax theorem, von Neumann (L2) stability.

## References

1. Conte, S. D. and deBoor, C., Elementary Numerical Analysis - An Algorithmic Approach, third edition, McGraw Hill, 1981.
2. Henrici, P., Elements of Numerical Analysis, John Wiley & Sons, 1964.
3. Froeberg, C. E., Numerical Mathematics - Theory and Computer Applications, The Benjamin Cummings Pub. Co., 1985.
4. Stoer, J. and Bulirsch, R., Introduction to Numerical Analysis, Texts in Applied Mathematics, Springer, 2002.
5. Press, William H.; Flannery, Brian P.; Teukolsky, Saul A. and Vetterling, William, T., Numerical Recipes in C: The Art of Scientific Computing, second edition, 1992.
6. Quarteroni, A.; Saleri, F. and Gervasio, P., Scientific Computing with MATLAB and Octave, third edition, Springer, 2010.
7. Thomas, J. W., Numerical Partial differential Equations: Finite Difference Methods, Springer, 1998.
8. Leveque, R. J., Numerical Methods for Conservation Laws, Lectures in Mathematics, ETH-Zurich, Birkhauser-Verlag, Basel, 1990.

## Course Plan

S.No	Session(s)	Topics to cover	Course Learning outcomes
1.	10	Unit-1	Solving an algebraic/transcendental equation to determine an approximate solution.
2.	7	Unit-2	Solving a system of linear equations to determine an approximate solution. Finding Approximate eigenvalues and an approximate inverse of a given matrix
3.	6	Unit-3	Construction of different types of interpolating polynomials satisfying the given data
4.	12	Unit-4	Evaluation of integrals numerically by Newton Cotes formulae and Gaussian quadrature and estimating the error involved in them.
5.	10	Unit-5	Solving ODE's numerically.
6.	10	Unit-6	Solving PDE's numerically.

## 2.23 Fluid Dynamics, MA 487/MM 569/ AM 569

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics/Applied Mathematics

**Course Code:** MA 487/MM 569/AM 569

**Title of the Course:** Fluid Dynamics

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Complex Analysis, Vector Calculus, Ordinary and Partial Differential Equations

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1 Define basic properties and laws that govern fluid motion, study kinematics and dynamics of fluids using these laws (Understand).
- CLO-2 Study the fluid properties and relationships between physical quantities (Understand)
- CLO-3 Understand the principles of conservation of mass, balance of momentum and energy applied to fluid motions (Understand).
- CLO-4 Formulate physical properties in the form of mathematical equations (Apply).
- CLO-5 Study the properties of fluids and the applications of the principles of fluid mechanics (Understand).
- CLO-6 Study different types of flows, their governing equations, examples illustrating such flows and the properties of physical quantities associated with such flows (Understand).

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	3	3	2	2	2	3	3	2
CLO2	3	3	2	2	2	3	3	2
CLO3	3	3	2	2	2	3	3	2
CLO4	3	3	2	2	2	3	3	2
CLO5	3	3	2	2	2	3	3	2
CLO6	3	3	2	2	2	3	3	2

## Syllabus:

- **Unit I:** Relation between cartesian coordinate systems to cylindrical polar coordinates and to spherical polar coordinates. Representation of vectors from one coordinate system to another coordinate system. Derivation of Curl, grad and divergence in cylindrical and spherical coordinate systems.
- **Unit II:** Continuum hypothesis, forces acting on a fluid, stress tensor, analysis of relative motion in the neighborhood of a point, Euler's theorem, equation of continuity, Reynolds transport theorem, conservation of mass, material surface, momentum equation.
- **Unit III:** Stream lines, Bernoulli's theorem, energy equation, circulation, Kelvin's circulation theorem, vorticity, Lagrange's theorem on permanence of vorticity, two dimensional irrotational flow of an incompressible fluid, Milne-Thomson circle theorem, Blasius' theorem, flow past an airfoil, the Joukowski transformation, theorem of Joukowski and Kutta.
- **Unit IV:** Axisymmetric flows, Stokes stream function, Butler's sphere theorem, flows due to source, doublet, uniform flow past a sphere, irrotational three dimensional flow, Weiss' sphere theorem. Constitutive equations for incompressible fluids, derivation of Navier-Stokes equations, unidirectional flows, Poiseuille flow, Couette flow, Stokes first and second problems, stagnation point flows, dynamical similarity and Reynolds number.
- **Unit V:** Flows at low Reynolds number, axisymmetric flow of a viscous fluid, uniform flow past a sphere at low Reynolds number, torque and drag on a sphere due to a uniform flow, Prandtl model for boundary layer, boundary layer equation, solution for a flow past a plate.

## References

1. Batchelor, G. K., An Introduction to Fluid Mechanics, Cambridge University Press, 1993.
2. Happel, J. and Brenner, H., Low Reynolds Number Hydrodynamics with Special Applications to Particulate Media, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1965.
3. Schlichting, H. and Gersten, K., Boundary-Layer Theory, with contributions by Egon Krause and Herbert Oertel, Jr. translated

from the ninth German edition by Katherine Mayes, eighth revised and enlarged edition, Springer-Verlag, Berlin, 2000.

4. Landau, L. D. and Lifshitz, E. M., Fluid Mechanics, Pergamon Press, London-Paris-Frankfurt; Addison-Wesley Publishing Co., Inc., 1959.
5. Kambe, T., Elementary Fluid Mechanics, World Scientific Publishing Co. Pvt. Ltd., Hackensack, NJ, 2007.
6. O'Neill, M. E. and Chorlton, F., Ideal and Incompressible Fluid Dynamics, Ellis Horwood Series: Mathematics and its Applications. Ellis Horwood Ltd., Chichester; Halsted Press (John Wiley & Sons, Inc.), New York, 1986.
7. Chorin, A. J. and Marsden, J. E., A Mathematical Introduction to Fluid Mechanics, third edition, Texts in Applied Mathematics, 4, Springer-Verlag, New York, 1993.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	10	Unit-I	Understand basic laws of orthogonal curvilinear coordinates and transformation of scalar and vector fields from one coordinate system to another.
2.	15	Unit-II	Understand the conservation laws and derive relevant mathematical equations
3.	15	Unit-III	Understand fluid properties related to vorticity and two dimensional flows
4.	15	Unit-IV	Understanding three dimensional flows, some singularity driven flows and the solutions of some special flows using Navier Stokes equations
5.	15	Unit-V	Understanding some problems in Stokes flows and boundary layer theory.



## 2.24 Classical Mechanics, MA 488/MM 570/AM 570

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics/Applied Mathematics

**Course Code:** MA 488/MM 570/AM 570

**Title of the Course:** Classical Mechanics

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Linear Algebra, Ordinary and partial differential equations

### **Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Explain the basic concepts of classical Mechanics, apply Newton's laws and write the kinetic energy and potential energy of a system in terms of generalized coordinates [Levels II, III].
- CLO-2: Distinguish the Lagrangian and Hamiltonian approaches to solve the equations of motion, rigid body dynamics [Level II].
- CLO-3: Use Poisson brackets to determine whether a given transformation is canonical or not [Levels II, III].
- CLO-4: Distinguish between the inertial and non-inertial frames of reference and write the rate of change of vector quantities in rotating frames [Level II].
- CLO-5: Explain Euler's theorems for the motions of rigid bodies [Level II].
- CLO-6: Compute moments of inertia for rigid bodies of various shapes [Level III].
- CLO-7 Analyze the motion of heavy symmetrical top with one fixed end point [Level IV].

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	3	1	1	3	2	3	2	2
CLO2	3	1	1	3	2	3	2	2
CLO3	3	1	1	3	2	3	2	2
CLO4	3	1	1	3	2	3	2	2
CLO5	3	1	1	3	2	3	2	2
CLO6	3	1	1	3	2	3	2	2
CLO7	3	1	1	3	2	3	2	2

### Syllabus:

- **Unit I:** Curvilinear co-ordinates : Cylindrical and spherical polar co-ordinates. Mechanics of a particle, mechanics of a system of particles, types of constraints, d'Alembert's principle,
- **Unit II:** Lagrange's equations, Lagrangian formulation in generalized co-ordinates, variational principles, Hamilton's principle of least action, derivation of Lagrange's equations from Hamilton's principle.
- **Unit III:** Legendre transformation, Hamiltonian, canonical equations, cyclic coordinates, Routh's procedure, generating functions, Poisson brackets, Liouville's theorem, infinitesimal canonical transformations, conservation theorems and angular momentum relations using Poisson brackets.
- **Unit IV:** Hamilton-Jacobi equations, Hamilton's Principal Function, example of Harmonic oscillator, Hamilton-Jacobi equation for Hamilton's Characteristic Function, action-angle variables in systems of one degree of freedom.
- **Unit V:** Central force problem: Equations of motion and first integrals, equivalent one dimensional problem, classification of orbits; Kepler's problem: inverse square law of force. Moving frames of reference: Non-inertial frames of reference, rate of change of a vector in a rotating frame, applications to particle kinetics, motion relative to earth, effects of Coriolis.
- **Unit VI:** Two dimensional problems in rigid body dynamics, examples. Kinematics of rigid body motion: Euler angles, Euler's theorem on the motion of a rigid body, infinitesimal rotations.

- **Unit VII:** The rigid body equations of motion : Angular momentum and kinetic energy of motion, inertia tensor and moment of inertia, inertial ellipsoid, the eigenvalues of the inertia tensor and the principal axis transformation, Euler's dynamical equations of motion under no external forces, torque-free motion of a rigid body, heavy symmetrical top with one point fixed.

## References

1. Goldstein, H., Poole, C. P. and Safko, J., Classical Mechanics, third edition, Pearson, 2011.
2. Chorlton, F., Textbook of Dynamics, second edition, Ellis Horwood Series: Mathematics and its Applications, Halsted Press (John Wiley & Sons, Inc.), New York, 1983.
3. Marion, J. B. and Thornton, S. T., Classical Dynamics of Particles and Systems, third edition, Harcourt Brace Jovanovich, 1988.
4. Scheck, Florian, Mechanics. From Newton's Laws to deterministic Chaos, fifth edition, Graduate Texts in Physics, Springer, Heidelberg, 2010.
5. Marsden, Jerrold E. and Ratiu, Tudor S., Introduction to Mechanics and Symmetry, A Basic Exposition of Classical Mechanical Systems, Texts in Applied Mathematics, 17, Springer-Verlag, New York, 1994.
6. Jose, Jorge V. and Saletan, E. G., Classical Dynamics, A Contemporary Approach, Cambridge University Press, 1998.

## Course Plan

S.No	Sessions	Topics to cover	Course Learning outcomes
1.	9	Unit - 1	The students learn to use curvilinear co-ordinates and learn about the motion of a single particle and system of particles and the various conservation laws.
2.	10	Unit - 2	The students learn about Lagrangian formulation and Hamilton's principle of least action
3.	10	Unit - 3	The students learn about Hamiltonian formulation and some important results pertaining to Hamilton's equations of motion using Poisson brackets
4.	5	Unit - 4	The students learn about Hamilton's Principle Function and Hamilton's characteristic function with examples
5.	3	Unit - 5	The students learn about equations of motion under central force motion and inverse square law and types of orbits.
6.	6	Unit - 6	The students learn about Non-Newtonian frames of reference and some examples
7.	12	Unit - 7	The students study rigid body dynamics. In particular they study important concepts like Euler angles and some important results using Moment of Inertia Tensor.

## 2.25 Commutative Algebra, MA 489/MM 551

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics

**Course Code:** MA 489/MM 551

**Title of the Course:** Commutative Algebra

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Algebra-I and Algebra-II.

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1 Employ basic operations like tensor product, localization to find out characteristics of given module. [Understand, Apply (Level II, Level III)]
- CLO-2 Understand concept related to primary decomposition. [Knowledge, Understand (Level I, Level II)]
- CLO-3 Visualize the extension of prime ideals in integral extension, particularly in the context of Dedekind domains. [Understand, Apply (Level II, Level III)]
- CLO-4 Understand underlying concepts behind Hilbert's Nullstellensatz. [Knowledge, Understand (Level I, Level II)]
- CLO-5 Gain a general idea, how the concepts of Commutative Algebra appear in Algebraic Number Theory and Algebraic Geometry. [Knowledge, Understand (Level I, Level II)]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2					2	2	2
CLO2	1						2	1
CLO3	2	2					2	2
CLO4	1						1	1
CLO5	1				1		2	1

## Syllabus:

- **Unit I:** Review : Rings (commutative rings with unity), ring homomorphism, ideals, quotient rings, prime ideals, maximal ideals, nil radical, Jacobson radical, extension and contraction of ideals, modules, module homomorphism, submodules, quotient modules, direct sum and direct product of modules, finitely generated modules, Nakayama lemma, exact sequence.
- **Unit II:** Tensor product : Tensor product of modules-construction and universal property, examples, associativity of tensor product, extension and restriction of scalars, compatibility of exact sequence and tensor product, flat modules and their characterization, faithfully flat modules, tensor product of algebras.
- **Unit III:** Localization & Primary decomposition : Multiplicatively closed sets, ring of fractions and module of fractions, examples, localization at a prime ideal, fraction operation on ideals, prime ideals in ring of fractions, compatibility of fraction operation with exact sequence, tensor product, local properties, primary ideals, primary decomposition, uniqueness theorems of primary decomposition.
- **Unit IV:** Integral extension : Integral element, ring of integers, integral extension, going-up theorem, integrally closed domain, going-down theorem, valuation rings, Hilbert's Nullstellensatz, Noether normalization lemma.
- **Unit V:** Noetherian and Artinian rings/modules : Ascending and descending chain conditions, Noetherian and Artinian rings/ modules and their basic properties, ring of fractions of Noetherian rings, Hilbert basis theorem, primary decomposition in Noetherian rings.
- **Unit VI:** Discrete valuation, Dedekind Domain & Dimension theory : Discrete valuation and discrete valuation rings, Noetherian integrally closed local domains, Dedekind domains, fractional ideals, factorization of ideals in Dedekind domain, Krull dimension of a ring, dimension of a module, Chevalley dimension, dimension theorem for finitely generated module over a Noetherian local ring, Krull's principal ideal theorem.

## References

1. Atiyah, M. F., and Macdonald, I. G., Introduction to Commutative Algebra, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1969.
2. Zariski, O. and Samuel, P., Commutative Algebra, Vol. I, GTM No.28, Springer-Verlag, New York, 1958.
3. Matsumura, Hideyuki, Commutative Algebra, second edition, Mathematics Lecture Note Series, 56, Benjamin/Cummings Publishing Co., Inc., Reading, Mass., 1980.
4. Gopalakrishnan, N. S., Commutative Algebra, Oxonian press, 1984.
5. Reid, M, Undergraduate Commutative Algebra, Cambridge University Press, 1995.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	5	Unit-I	Define Rings (commutative rings with unity), ring homomorphism, ideals, quotient rings and give examples Define prime ideals, maximal ideals, nil radical, Jacobson radical, extension and contraction of ideals, give examples and solve basic problems. Define modules, module homomorphism, submodules, quotient modules, direct sum and direct product of modules, give examples and perform basic operations on modules Define finitely generated modules, prove Nakayama lemma, give applications, define exact sequence, give examples.
2.	5	Unit-II	Define tensor product of modules, prove universal property, give examples, prove associativity of tensor product Define extension and restriction of scalars, determine compatibility of exact sequence and tensor product. Define flat modules and prove their characterization, give examples, define faithfully flat modules, define tensor product of algebras, solve basic problems related to tensor product, exact sequence, flat and faithfully flat module.
3.	10	Unit-III	Define multiplicatively closed sets, ring of fractions and module of fractions, give examples, in particular, localization at a prime ideal, observe fraction operation on ideals, identify prime ideals in ring of fractions. Examine compatibility of fraction operation with exact sequence, tensor product, define local property, give examples, solve basic problems.
4.	10	Unit-IV	Define Integral element, ring of integers, give examples, define integral extension, prove going-up theorem. Define integrally closed domain, give examples, prove going-down theorem. Define valuation rings, give examples, prove Hilbert's Nullstellensatz, prove Noether normalization lemma, give applications.
5.	10	Unit-V	Define Ascending and descending chain conditions, give examples, define Noetherian and Artinian rings/modules, give examples and exhibit their basic properties. Study ring of fractions of Noetherian rings, prove Hilbert basis theorem, understand primary decomposition in Noetherian rings, give applications, solve basic problems.
6.	15	Unit-VI	Define Krull dimension of a ring, give example, define dimension of a module and Chevalley dimension, calculate in terms of basic examples. Prove dimension theorem for finitely generated module over a Noetherian local ring, give applications, prove Krull's principal ideal theorem as a corollary.

## 2.26 Graph Theory and Algorithms, MA 491/MM 581/AM 581

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics/Applied Mathematics

**Course Code:** MA491/MM 581/AM 581

**Title of the Course:** Graph Theory & Algorithms

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Complex Analysis, Vector Calculus, Ordinary and partial differential equations

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1 Learn basic notions such as Paths, Cycles and Spanning Trees [Level I, Level II]
- CLO- 2 Learn about connectivity of graphs and solve shortest path problems [Level II, Level III]
- CLO- 3 Solve Hall's Marriage problem (Matchings)[Level I, Level III, Level IV ]
- CLO- 4 Learn about Eulerian tours and solve Chinese postman problem, Hamiltonian graphs and solve Traveling salesman problems [Level II, Level III, Level IV]
- CLO- 5 Understand Four Colour problem and planar graphs [Level II, Level IV]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	1						1	
CLO2	2					1	1	
CLO3	2					1	1	1
CLO4	2					2	1	2
CLO5	1						1	2

## Syllabus:

- **Unit I:** Introductions to Graphs  
Vertex Degree, Hand Shake Lemma, Subgraphs, isomorphism of graphs, Paths, Cycles and matrix representations of graphs, Trees, Spanning Trees, vertex connectivity
- **Unit II:** Connectivity problems  
Shortest path problems (Breadth first Search algorithm), Minimal spanning tree problem (Kruskal's and Prim's algorithms), Dijkstra's algorithm in weighted graphs, with proofs of validity of each algorithms, Whitney's theorem ( 2-connectivity)
- **Unit III:** Euler and Hamiltonian graphs  
Euler's tours and Hamiltonian cycles in graphs, Necessary and sufficient conditions for a graph to have Euler tour and Hamiltonian Cycles, Fluery's algorithm, Chinese postman Problems and Traveling Salesman Problems
- **Unit IV:** Matchings in Graphs  
Matchings and M-Aurmenting Paths, Hall's Marriage Theorem, Tute's 1-factor theorem, Hungarian algorithm (maximal matching), Personal & Optimal Assignment Problems, Kuhn-Munkres Algorithm, general Chinese Postman problem
- **Unit V:** Planar Graphs and Colouring  
Euler's formula, classification of Platonic bodies, Kuratowski's theorem, Vertex and Edge colouring, Four Colour Problem
- **Unit VI:** Directed graphs and Networks  
Indegree and Outdegree of a vertex, Tournaments, Network flows and cuts, Max-Flow-Min-Cut Theorem and Mengers theorem

## References

1. Harary, F., *Graph Theory*, Addison-Wesley Publishing Co., London, 1969.
2. J Clark and D A Holton, *A First Look at Graph Theory*, World Scientific Publ. Co. Allied Publishers 1995
3. Gibbons, A., *Algorithmic Graph Theory* Cambridge University Press
4. R Diestel, *Graph Theory*, GTM 173 Springer 2010



## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	8	Unit-I	Review of basics of Graph theory
2.	12	Unit-II	To be able to solve shortest distance problems using various algorithms
3.	10	Unit-III	To solve CPP and Traveling Salesman Problems
4.	10	Unit-IV	To be able to solve Optimal/Personal Assignment problems
5.	8	Unit-V	Classification of platonic boides
5.	7	Unit-VI	To have knowledge of Networks and applications

## 2.27 Lie Algebras, MA 479/MM 573/AM 573

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics/Applied Mathematics

**Course Code:** MA 479/MM 573/AM 573

**Title of the Course:** Lie Algebras

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Nil

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1 : Learn basics of Lie algebras, solvable and nilpotent Lie algebras (Apply)
- CLO-2 : Learn semi simple Lie algebras, Cartan's criterion, connection between solvable and semi simple Lie algebras, classification of  $SL(2)$  modules, Weyl's theorem on complete reducibility (Apply)
- CLO-3 : Learn notion of total and Cartan sub algebras, using Cartan sub algebras slowly lead to root systems. (Apply)
- CLO-4 : Learn basics of root systems, Weyl group combinatorics (Apply).
- CLO-5 : Study Classification of root systems leading to classification of simple Lie algebras (Apply)
- CLO-6 : universal enveloping algebra definition, statement of Serre's theorem , basics of Verma modules and statement of classification of finite dimensional irreducible representations of semi simple Lie algebras. (Comprehend)

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2			2	2		2	2
CLO2	2			2	2		2	3
CLO3	2			2	2		2	3
CLO4	2			2	2		2	3
CLO5	2			2	2		2	3
CLO6	2			2	2		2	3

## Syllabus:

- **Unit I:** Definition of Lie algebras, classical examples, ideals, standard isomorphism theorems, nilpotent Lie algebras, solvable Lie algebras, simple Lie algebras, Engel's theorem.
- **Unit II:** Lie's theorem, Jordan decomposition, Cartan's criterion for solvability, Cartan-Killing form, semisimplicity,  $SL(2)$  representations.
- **Unit III:** Total and Cartan subalgebras leading to root systems.
- **Unit IV:** Learn definition of root systems, basics, Weyl group, properties.
- **Unit V:** Study of simple root systems, Weyl group, simple root systems, Dynkin diagrams, classification of simple root systems,
- **Unit VI:** universal enveloping algebra, statement of PBW theorems, roots and weights calculation in classical set up, statement of Serre's theorem, definition and basic properties of Verma modules, statement of classification of finite dimensional representations of simple Lie algebras.

## References

1. Humphreys, J., Introduction to Lie Algebras and Representation Theory, GTM 9, Springer-Verlag, 1972.
2. Serre, Jean-Pierre, Complex Semisimple Lie Algebras, Springer Monographs, 2001.

## Course Plan

S. No.	Sessions	Topics to cover	Learning outcomes
1.	11	Unit-I	Students should study basics of Lie algebras and apply it to solve problems.
2.	11	Unit-II	Students should study various Lie algebras and apply it to solve problems.
3.	8	Unit-III	Students should study total and cartan subalgebras and apply it to solve problems.
4.	9	Unit-IV	Students should study root systems and apply it to solve problems.
5.	8	Unit-V	Students should study Dynkin diagrams and apply it to solve problems.
6.	8	Unit-VI	Students should learn about Verma modules and irreducible modules.

## 2.28 Algebraic Geometry, MA 493/MM 578

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics/Applied Mathematics

**Course Code:** MA 493/MM 578

**Title of the Course:** Algebraic Geometry

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Algebra-I and Algebra-II courses of M.Sc. Mathematics curriculum.

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

CLO-1 Understand the basic theory in Algebraic geometry. [Knowledge, Understand(Level I, Level II)]

CLO-2 Understand and solve few basic problems. [Knowledge, Understand(Level I, Level III)]

CLO-3 Apply algebraic concepts to understand geometry. [Knowledge, Understand(Level I, Level III)]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	3			3	3		3	3
CLO2	3		3	3	3		3	3
CLO3	3	3	3	3	3		3	3

### Syllabus:

- **Unit I:** Commutative algebra: Localization, dimension theorem for Noetherian rings (without proof), Rings of dimension 0, 1, normal Noetherian rings of dimension 1 (i.e., Dedekind domains), normalization lemma and Hilbert's null stellensatz (without proof), transcendental extensions and Luroth's theorem.

- **Unit II:** Geometric concepts: Prime spectrum of a commutative ring with Zariski topology, irreducible algebraic sets and affine algebraic varieties, criterion for connectedness of affine algebraic sets; Noetherian topological spaces, principle of Noetherian induction and application to algebraic varieties (decomposing an algebraic variety into a finite union of irreducible components); projective spectrum of a polynomial ring and projective varieties.
- **Unit III:** Affine plane curves: Classification of algebraic subsets of the plane, degree of a plane curve, intersection of curves (via dimension theorem and also via elimination theory) and weak form of Bezout's theorem; regular and rational functions on a curve.
- **Unit IV:** Rational curves: Rational and non-rational curves, conics, a characterization of rational curves in terms of the field of rational functions on the curve, birational isomorphisms. Projective plane curves: Projective completion of an affine curve, homogenization and dehomogenization, resultant of homogeneous polynomials.
- **Unit V:** Analysis of singularities: Order of contact of a line with a curve at a point, multiplicity of a point, smooth and multiple points, tangent space and tangent cone at a point, simple and multiple tangents, ordinary multiple points, finiteness of the singular locus of a curve; characterization of smooth points and  $r$ -fold points, Sezout's theorem.

## References

1. Miles, Reid, Undergraduate Algebraic Geometry, Student Text Books, London Mathematical Society Student Texts (Book 12), Cambridge University Press, 1989.
2. Walker, R. J., Algebraic Curves, Springer-Verlag, Berlin-New York, 1978.
3. Hartshorne, R., Algebraic Geometry, Issue 52 of Graduate Texts in Mathematics, Lecture notes in mathematics, Volume 687, Springer, 1977.
4. Shafarevich, I. R., Basic Algebraic Geometry, GTM, Springer, second revised and expanded edition, 1994.
5. Atiyah, M. F. and Macdonald, I. G., Introduction to Commutative Algebra, Addison-Wesley Publishing Co., 1969.

6. Fulton, W., Algebraic Curves, An Introduction to Algebraic Geometry, Notes written with the collaboration of Richard Weiss, Mathematics Lecture Notes Series, W. A. Benjamin, Inc., New York-Amsterdam, 1969.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	10	Unit-I	Understand fundamental concepts. Able to relate and understand between the dimension and transcendental degree.
2.	15	Unit-II	Able to understand geometry with algebraic objects.
3.	15	Unit-III	We apply theory with plane curves(affine)
4.	15	Unit-IV	We apply theory with plane curves(projective) We are able to relate geometric concepts like tangent space and its dimension, regular/smooth point with algebra.

## 2.29 Topology-II, MA 482/MM 577/AM 577

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics or/and Applied Mathematics

**Course Code:** MA 482/MM 577/AM 577

**Title of the Course:** Topology-II

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):**

This course is an elective course for M.Sc. Mathematics and Applied Mathematics students. Those students who have completed Topology-I (code MM453) and Algebra-I (code MM403) are eligible for taking this course.

**Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1 Understand the homotopy theory. [Level I, II]
- CLO-2 Classify a few topological spaces up to the homotopy equivalence. [Level II, IV]
- CLO-3 Calculate the fundamental groups of topological spaces. [Level III]
- CLO-4 State and prove the Van Kampen theorem and understand CW-complex. [Level I, II]
- CLO-5 Construct a topological space whose fundamental group is the given group. [Level VI]
- CLO-6 Understand the covering spaces and examples. [Level II, III]
- CLO-7 Apply covering theory to the lifting problems, a few questions in complex analysis. [Level III]
- CLO-8 Define various concepts from homological algebra. [Level I]
- CLO-9 Calculate different homologies for some topological space. [Level IV]
- CLO-10 A student should get familiar with the category theory. [Level I]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2		3	3	3	2	3	2
CLO2	3	3	3	3	3		3	2
CLO3	3	3	3	3	3	3	3	2
CLO4							3	
CLO5	3		3	3	3		3	3
CLO6			2			3	3	3
CLO7	3	2	3	3			3	3
CLO8	3						3	3
CLO9	3	3	3	3	3		3	3
CLO10	2		2	2	2	3	3	2

**Syllabus:**

- **Unit I:** (The Fundamental groups and homotopy theory): homotopy mappings, contractible spaces, homotopically equivalent spaces, fundamental group of topological spaces, homotopy invariance of the fundamental group, the fundamental group of a circle is isomorphic to the group of integers. The Brower's fixed point theorem, the fundamental theorem of algebra, introduction to higher homotopy groups.
- **Unit II:** (The category theory): category theory and the functors between the categories, the fundamental group functor.
- **Unit III:** (The Van Kampen theorem and CW-complexes): Finding a topological space for a given group as a fundamental group, Van Kampen theorem to calculate the fundamental group of a topological space when fundamental groups of subspaces are given.
- **Unit IV:** (Covering space theory): definition of a covering Space and examples of covering spaces. Properties of covering spaces, the relation between the fundamental group of covering space and its base, the lifting criteria, universal covering space construction.
- **Unit V:** (Simplicial theory): Various notions like simplicial complexes, barycentric subdivision, simplicial maps, approximation theorem, the fundamental group of a simplicial complex.



## References

1. Bredon, G. E., Topology and Geometry, Graduate Texts in Mathematics 139, Springer-Verlag, 1997.
2. Hatcher, A., Algebraic Topology, Cambridge University Press, 2002.
3. Hocking, J. G. and Young, G. S., Topology, second edition, Dover Publications, New York, 1988.
4. Greenberg, M. J. and Harper, J. R., Algebraic Topology: A First Course, Mathematics Lecture Note Series 58, Benjamin/Cummings Publishing Co., 1981.
5. Massey, W. S., A Basic Course in Algebraic Topology, Graduate Texts in Mathematics 127, Springer-Verlag, 1991.
6. Maunder, C. R. F., Algebraic Topology, Cambridge University Press, 1980.

## Course Plan

S.No	Sessions	Topics to cover	CLOs
1.	15	Unit-I	Understand basics of homotopy mappings, homotopy equivalence and fundamental groups of topological spaces
2.	5	Unit-II	Understand basics of category theory with view towards its application in understanding the fundamental group functor
4.	10	Unit-III	Understand Van Kampen theorem and CW-complex use of these result/construction to obtain the fundamental group of some topological spaces
5.	15	Unit-IV	Understand the properties of covering space of a topological space and to find a relation between the fundamental group of a topological space and its covering space.
6.	10	Unit-V	Understand basics of simplicial complexes.

## 2.30 Continuum Mechanics, AM 586/MM 586

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics/Applied Mathematics

**Course Code:** AM 586/MM 586

**Title of the Course:** Continuum Mechanics

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):**

Vector Analysis, Cartesian tensors, Linear algebra, Differential Equations

**Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1 Describe motion, deformation and forces in a continuum (understand).
- CLO-2 Derive equations of motion and conservation laws for a continuum (apply).
- CLO-3 Learn the constitutive equations for solid and fluids (understand).
- CLO-4 Solve simple boundary value problems for solids and fluids (Apply)
- CLO-5 Apply the conservation principles and the constitutive equations (Apply).

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	3	2	1	2	2	3	3	2
CLO2	3	2	1	2	2	3	3	2
CLO3	3	2	1	2	2	3	3	2
CLO4	3	2	1	2	2	3	3	2
CLO5	3	2	1	2	2	3	3	2

## Syllabus:

- **Unit I:** Cartesian tensors. Description of continua and kinematics.
- **Unit II:** Forces in a continuum. The polar decomposition theorem. Continuum deformation.
- **Unit III:** Geometrical restrictions on the form of constitutive equations. Constitutive equations for fluid, elastic and thermo-elastic materials.
- **Unit IV:** Shear flow solutions of Reiner-Rivlin fluids.
- **Unit V:** Some solutions of the Navier-Stokes equations.
- **Unit VI:** General theorems in inviscid hydrodynamics.

## References

1. Hunter, S. C., Mechanics of continuous Media, Mathematics and its Applications. Ellis Horwood Ltd., Chichester Halsted Press, (John Wiley and Sons Inc.), New York-London-Sydney, 1976.
2. Chadwick, P., Continuum Mechanics: Concise Theory and Problems, Dover, second edition, Dover Publications, Inc., Mineola, NY, 1999.
3. Lai, W. M., Rubin, D. and Krempf, E., Introduction to Continuum Mechanics, third edition, Butterworth Heinemann Ltd., 1993.
4. Sedov, L. I., A Course in Continuum Mechanics, Wolters-Noordhoff, Groningen, 1971.
5. Narasimhan, M. N. L., Principles of Continuum Mechanics, Wiley, New York, 1993.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	5	Unit-I	The student will be able to understand about general stresses and deformations in continuous materials.
2.	5	Unit-II	Students should study continuous functions and apply it to solve problems.
3.	10	Unit-III	The student will learn about the constitutive equations for fluid, elastic and thermo-elastic materials.
4.	10	Unit-IV	The students learn about some Reiner Rivlin fluids
5.	10	Unit-V	The students learn about some analytical solutions of Navier-Stokes equations
6.	15	Unit-VI	The students learn about a few important theorems pertaining to inviscid fluid flows.

## 2.31 Algebraic Number Theory, MA 484/MM 585

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics/Applied Mathematics

**Course Code:** MA 484/MM 585

**Title of the Course:** Algebraic Number Theory

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):**

Introduction to Number Theory (MM 506) and Algebra III (MM 505) (some part of Galois Theory) are prerequisites for this course.

**Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Understand integral extensions of rings with examples [Level I, Level II]
- CLO-2: Study the Ring of Algebraic Integers in Number Field [ Level I, Level II]
- CLO- 3: Understand ideal factorizations using Dedekind Domain Structure and some computations [Level II, Level III]
- CLO- 4: Find integral basis [Level I, II]
- CLO- 5: Understand finiteness of Ideal Class Group [Level I]
- CLO- 6: Understand structure of units in  $\mathcal{O}_K$  [Level I,II]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	1						1	
CLO2	1						2	
CLO3		2					2	3
CLO4	2						2	2
CLO5							2	3
CLO6							2	3

## Syllabus:

- **Unit I:** Introduction  
Motivation, Gaussian integers  $\mathbb{Z}[i]$  and the ring  $\mathbb{Z}[\sqrt{-5}]$ , Classification of Gaussian Primes, Ideals and Lattice structures in these rings
- **Unit II:** The Ring of Algebraic Integers  $\mathcal{O}_K$   
Integral Elements and equivalent conditions for being integral, integral ring extensions, Integral closure, The Ring of algebraic integers of a number field  $\mathcal{O}_K$ , Integral basis Theorem : existence and Construction, Trace and Norm maps:  $Tr_{K/\mathbb{Q}}$  and  $N_{K/\mathbb{Q}}$ , Discriminants, Group of Units in Imaginary Quadratic fields, Cyclotomic Integers
- **Unit III:** Dedekind Domains  
Structure of the ring of integers  $\mathcal{O}_K$ : Noetherian, integrally closed and every prime ideal is maximal, Fractional ideals, Dedekind domains and equivalence properties, Uniqueness of Factorization of ideals into product of prime ideals, Ramification of Primes (in Quadratic Number Fields)
- **Unit IV:** The Ideal Class Group  
Definition of Ideal Class Group, Finiteness of Class Group: Minkowski's Theorem (on lattice points) and applications
- **Unit V:** Units in the Ring of Integers Group of Units in  $\mathcal{O}_K$ , The Dirichlet's unit theorem (with proof)

## References

1. TIFR Mathematical Pamphlet: Algebraic Number Theory (online)
2. Artin, M., Algebra, Prentice Hall of India, 1991.
3. Serre, J.P., Local Fields, Springer, Graduate Texts in Math, 1995.
4. J S Milne, Algebraic Number Theory (online)
5. Rosen, M. and Ireland, K., A Classical Introduction to Number Theory, GTM, Springer, 1982.
6. Ian Stewart and David Tall, Algebraic Number Theory and Fermat's Last Theorem, Taylor & Francis

## Course Plan

S. No	Sessions	Topics	Course Learning Outcomes
1.	8	Unit-I	To understand problems in factorizations
2.	16	Unit-II	To study structural properties of $\mathcal{O}_K$ , construction of integral basis, discriminants and ramification of primes
3.	10	Unit-III	To understand Ideal factorization and its uniqueness
4.	10	Unit-IV	To study finiteness of Ideal Class Group
5.	10	Unit-V	To understand structure of group of units in $\mathcal{O}_K$

## 2.32 Mathematical Logic, MA 490/IM 457/MM 592/AM 592/ST 592

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics or/and Applied Mathematics/Statistics

**Course Code:** MA 490/IM 457/MM 592/AM 592/ST 592

**Title of the Course:** Mathematical Logic

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** UG level Mathematics

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1: Get some exposure to the foundations of mathematics [Level I, Level II].
- CLO-2: Understand the logic of propositions and analyze various aspects of propositional formulae [Level I, Level IV].
- CLO-3: Determine the need for axiomatic approach to propositional logic and prove soundness and completeness theorems [Level I, Level II].
- CLO-4: Understand the first order logic as generalization of propositional logic and analyze various aspects of the languages associated with structure and formulae [Level I, Level IV].

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2	1	1	1	3	1	2	1
CLO2	2	1	2	3	3	1	2	1
CLO3	3	1	2	3	3	1	2	1
CLO4	3	1	3	2	3	1	2	1

## Syllabus:

- **Unit I:** Classical Propositional Calculus(PC): Syntax- Propositional formulae, height and length of formula, proof by induction on the set of formulae, the decomposition tree of a formula, the uniqueness decomposition theorem, substitution in a formula, Valuations and truth tables, Truth functions, Logical equivalence relation.
- **Unit II:** Semantic consequence and satisfiability. Compactness theorem with application. Adequacy of connectives. Normal forms.
- **Unit III:** Axiomatic Approach to PC: Sequent calculus, Soundness, consistency, completeness
- **Unit IV:** Classical First Order Logic(FOL) and First Order Theories: Syntax, Satisfaction, truth, validity in FOL.
- **Unit V:** Axiomatic approach, soundness. Consistency of FOL and completeness (sketch), Equality, Examples of first order theories with equality.

## Reference/Reading Material:

1. R. Cori and D. Lascar: Mathematical Logic, Oxford, 2001.
2. Arindama Singh. Logics for Computer science, PHI Learning, Second Edition 2018.
3. I. Chiswell and W. Hodges: Mathematical Logic. Oxford, 2007.
4. J. Kelly: The Essence of Logic, Pearson, 2011.
5. A. Margaris, First Order Mathematical Logic, Dover, 1990.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	10	Unit-I	Learn classical propositional logic
2.	5	Unit-II	Understand consequence and normal forms
3.	4	Unit-III	Understand axiomatic scheme and completeness of propositional logic
4.	10	Unit-IV	Learn first order logic and related concepts
5.	12	Unit-V	Learn axiomatic scheme and completeness of first order logic



## 2.33 Dynamical Equations on time scales, MA 472/MM 597/AM 597

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics or/and Applied Mathematics

**Course Code:** MA 472/MM 597/AM 597

**Title of the Course:** Dynamical Equations on time scales

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Differential Equations

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1 Understand the time scale calculus such as continuity, differentiation, integration etc. and basic results related to this. [Understand]
- CLO-2: Understand the first and second order dynamic equations on time scales and find its solutions by using different techniques. [Understand, Apply]
- CLO-3: Apply time scale calculus to study the results related to BVPs, eigenvalue problems, and Green's functions. [Apply]
- CLO-4: Understand the solutions of the linear systems and higher order equations and analyze qualitative behavior of the solutions. [Understand]
- CLO-5: Derive the different inequalities by using time scale calculus. [Apply]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2						3	3
CLO2	2		2				3	3
CLO3	2		2				3	3
CLO4	2		2				3	3
CLO5	2		3				3	3

## Course syllabus

- **Unit I:** The Time Scale Calculus : Basic definitions, Differentiation, Examples and applications, Integration, Chain rule, Polynomials, Basic results.
- **Unit II:** First Order Linear Equations: Hilger's complex plane, Exponential functions and examples, Initial value problems.
- **Unit III:** Second Order Linear Equations : Wronskians, Hyperbolic and Trigonometric functions, Reduction of order, Method of factoring, Non-constant coefficients, Euler-Cauchy Equations, Variation of parameters, Annihilator method, Laplace transform.
- **Unit IV:** Self Adjoint Equations : Preliminaries and examples, Riccati equation, Disconjugacy, Boundary value problems and Green's function, Eigenvalue problems.
- **Unit V:** Linear Systems and Higher Order Equations : Regressive matrices, Constant coefficients, Self-adjoint Matrix equations, Asymptotic behavior of solutions, Higher order linear dynamic equations.
- **Unit VI:** Dynamic Inequalities: Gronwall's inequality, Holder's and Minkowski's inequalities, Jensen's inequality, Opial's inequalities, Lyapunov inequalities, Upper and Lower solutions.

## References

1. Martin Bohner and Allan Peterson, Dynamic Equations On Time Scales : An Introduction With Applications, Birkhauser, 2001.
2. Martin Bohner and Allan Peterson, Advances in Dynamic Equations On Time Scales, Birkhauser, 2002.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	15	Unit-I	Understand the time scale calculus
2.	10	Unit-II	Understand the first and second order dynamic equations on time scales
3.	10	Unit-III	Apply Time scale calculus to BVPs, Eigen value problems, and Greens functions
4.	10	Unit-IV	Understand the solutions of Linear systems , higher order equations and qualitative theory
5.	10	Unit-V	Derive the different inequalities on different time scales

## 2.34 Functional Differential Equations, MA 471/MM 596/AM 596

**Name of the Academic Program:** I.M.Sc. in Mathematical Sciences and M.Sc in Mathematics/Applied Mathematics

**Course Code:** MA 471/MM 596/AM 596

**Title of the Course:** Functional Differential Equations

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** Differential Equations, Linear Algebra

### **Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Understand the difference and differential equations and neutral difference equations. [Understand]
- CLO-2: Understand the neutral functional differential equations and its basic theory, existence and uniqueness of solution of these problems and apply the some standard technique to find the solution of a given retarded functional differential equation. [Understand, Apply]
- CLO-3: Understand the properties of the solution map and Autonomous and periodic process. [Understand]
- CLO-4: Understand the stability theory of functional differential equations and use this concept to classify the solutions by using the Lyapunov function. [Understand, Apply]
- CLO-5: Understand the linear systems and neutral differential equations and its applications. [Understand , Apply]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2		2				3	3
CLO2	2		2				3	3
CLO3	2		2				3	3
CLO4	2		2				3	3
CLO5	2		2				3	3

### Course syllabus

- **Unit I:** Linear differential difference equations : Differential and difference equations, Retarded differential difference equations, Exponential estimates, Characteristic equation, Fundamental solution, Variation-of-constants formula, Neutral differential difference equations.
- **Unit II:** Retarded functional differential equations basic theory: Definition, Existence, uniqueness, and continuous dependence, Continuation of solutions, Differentiability of solutions, Backward continuation, Caratheodory conditions.
- **Unit III:** Properties of the solution map : Finite-or infinite-dimensional problem; Equivalence classes of solutions, Exponential decrease for linear systems, Unique backward extensions, Range in  $\mathbb{R}$ , Compactness and representation.
- **Unit IV:** Autonomous and periodic processes : Processes, Invariance, Discrete systems maximal compact invariant sets, Fixed points of discrete dissipative processes, Stability and maximal invariant sets in processes , Periodic trajectories of a periodic processes, Convergent systems.
- **Unit V:** Stability theory : Definitions, The method of Lyapunov functional, Lyapunov functional for autonomous systems, Razumikhin-type theorems.
- **Unit VI:** General linear systems : Global existence and exponential estimates, Variation-of-constants formula, The formal adjoint equation, The true adjoint Boundary-value problems, Stability and boundedness; Neutral differential equations and applications.

### References

1. J. K. Hale, Theory of Functional Differential Equations, 1977.

2. V. B. Kolmanovskii, V. R. Nosov, Stability of Functional Differential Equations, Academic Press, 1986.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	13	Unit-I	Understand the difference and differential equations
2.	12	Unit-II	Understand the theory of functional differential equations
3.	10	Unit-III	Understand the solution map, Autonomous and periodic process
4.	10	Unit-IV	Understand the stability theory FDE and classify the solutions by using Lyapunov function
5.	10	Unit-V	Understand the neutral differential equations and its applications

## 2.35 Linear Algebra and Matrix Theory, ST 302/ST 402

**Course Code:** (ST 302/ST 402): IMSc in Mathematical Sciences and MSc in Statistics-OR

**Title of the Course:** Linear Algebra and Matrix Theory

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** This is a course which helps students to visualize vector spaces especially real vector spaces, distances, and angles therein. Further it imparts knowledge on Linear Algebra and Matrices that is required for studying and proving results in Multivariate Distributions, Hypothesis testing in Design of Experiments, Regression Analysis.

**Pre-requisite Course/Knowledge (if any):** UG-Mathematics

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Visualize real vector spaces of higher but finite order, distances between vectors and angles between them via inner product.
- CLO-2: Explain linear independence, dimension of a vector space, Linear spans of collection of vectors and display subspaces, .
- CLO-3: Apply the concepts of Linear Transformations on vectors and determine if exact solutions to systems of linear equations with multiple dimensions/ variables exist or not and determine if they do.
- CLO-4: Work with real matrices: Understand what column spans and row spans of matrices are and prove results on Eigenvalues and quadratic forms.
- CLO-5: Explain what Generalized inverses of matrices are and prove results and solve problems where these are needed.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	3							3			3		
CLO-2	3							3			3		
CLO-3	3							3			3		
CLO-4	3							3			3		
CLO-5	3							3			3		
CLO-6	3							3			3		

### Syllabus

- Unit I:** System of linear equations, solutions of systems of linear equations, homogeneous system, Matrices: Elementary operations, reduced row-echelon form, Concept of fields with examples, Vector spaces and Subspaces with examples, Direct sum and Algebra of subspaces viz. sum, intersection, union etc, Linear combinations, Spanning sets, Linear spans, Linear dependence and independence in vector spaces, Row and Column space of a matrix, Basis and Dimensions.
- Unit II:** Linear Transformations, Kernel and Image of a linear transformation, Rank and Nullity, Matrix representation of a linear operator, Change of Basis, Similarity, Dual space, Inner product spaces with examples, Cauchy-Schwarz inequality with applications, Orthogonality, Orthonormal sets and Bases, Gram Schmidt Orthogonalization Process, adjoint operator.
- Unit III:** Eigenvalues and eigenvectors, Cayley Hamilton theorem, Algebraic and geometric multiplicity of characteristic roots, Diagonalization of matrices, Determinants, Relating trace and determinant with eigenvalues, Spectral decomposition of a symmetrical matrix (Full rank and non-full rank cases), Example of spectral decomposition, Spectral decomposition of asymmetric matrix, Factorization of a matrix, Eigenvalues and eigenvectors for solution of Differential equations.
- Unit IV:** Generalized inverse of a matrix, Different classes of generalized inverse, Properties of g-inverse, Reflexive g-inverse, left weak and right weak g-inverse, Moore- Penrose (MP) g-inverse and its properties, Real quadratic forms, Linear transformation of quadratic forms, Index and signature, Reduction of quadratic form into sum of squares, Gram matrix with examples, Jordan canonical form.

## References / Reading Material :

1. Biswas, S., A Text Book of Matrix Algebra, 2nd ed., New Age International Publishers, 1997.
2. Golub, G.H. and Van Loan, C.F., Matrix Computations, 2nd ed., John Hopkins University Press, Baltimore-London, 1989.
3. Hadley, G., Linear Algebra, Narosa Publishing House (Reprint), 2002.
4. Hoffman, K. and Kunze, R., Linear Algebra, second edition, Prentice-Hall, New Delhi, 1978.
5. Robinson, D.J.S., A Course in Linear Algebra with Applications, World Scientific, Singapore, 1991.
6. Rao, C.R., Linear Statistical Inferences and its Applications, 2nd ed., John Wiley & Sons, 1973.
7. Searle, S.R., Matrix Algebra useful for Statistics, John Wiley & Sons, 1982.
8. Strang, G., Linear Algebra and its Application, 2nd ed., Academic Press, London New York, 1980.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	System of equations, solutions of systems of equations, homogeneous system, Matrices: Elementary operations, reduced row-echelon form.	3
Week 2	I	Concept of groups and fields with examples, Vector spaces and Sub-spaces with examples, Direct sum of subspaces. Algebra of subspaces viz. sum, intersection, union etc.	1
Week 3	I	Row and Column space of a matrix, Linear combinations, Spanning sets, Linear spans, Linear dependence and independence in vector spaces, Basis and Dimensions.	3
Week 4	I	Linear Transformations, Kernel and Image of a linear transformation, Rank and Nullity, Matrix representation of a linear operator.	3
Week 5	II	Change of Basis, Similarity, Dual space.	2
Week 6	II	Inner product spaces with examples, Cauchy-Schwarz inequality with applications, Orthogonality, Orthonormal sets and Bases.	1
Week 7	II	Gram Schmidt Orthogonalization Process, Adjoint operator.	1
Week 8	II	Eigenvalues and eigenvectors, Cayley Hamilton theorem, Algebraic and geometric multiplicity of characteristic roots.	4
Week 9	III	Diagonalization of matrices, Determinants, Relating trace and determinant with eigen values.	3
Week 10	III	Spectral decomposition of a symmetrical matrix (Full rank and non-full rank cases), Example of spectral decomposition, Spectral decomposition of asymmetric matrix, Factorization of a matrix, Eigenvalues and eigenvectors for solution of Differential equations.	4
Week 11	III	Matrix decomposition. Singular value decomposition, LU-decomposition, QR-decomposition, Cholesky decomposition	4
Week 12	III	Definiteness-positive, non-negative of real matrices, g-inverses: existence and definition,	5
Week 13	IV	Generalized inverse of a matrix, Different classes of generalized inverse, Properties of g-inverse, Reflexive g-inverse, left weak and right weak g-inverse, Moore- Penrose (MP) g-inverse and its properties, form.	5
Week 14	IV	Real quadratic form, Linear transformation of quadratic forms, Index and signature, Reduction of quadratic form into sum of squares, Gram matrix with example, Jordan canonical.	5



## 2.36 Statistical Methods, ST 305/ST 407

**Course Code:** (ST 305/ST 407): IMSc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Statistical Methods

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** Based on quantitative methods in Statistics, objective is to enhance the practical knowledge of an individual in Statistical problem solving with help of Computer Software.

**Pre-requisite Course/Knowledge (if any):** Basic Mathematics, UG-Statistics

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Obtain and interpret graphical and Diagrammatic representation of data.
- CLO-2: Obtain and interpret summary measure of univariate, bivariate, multivariate data.
- CLO-3: Discuss the importance of statistical techniques and concepts in the different areas of applied statistics.
- CLO-4: Apply parametric and non-parametric statistical techniques in decision making.
- CLO-5: Simulate appropriate random observations.
- CLO-6: Carry out basic, non-linear, non-parametric, polynomial regression to any given sample of data.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	3		3	3							3		
CLO-2	3		3	3							3		
CLO-3	3		3	3							3		
CLO-4	3		3	3							3		
CLO-5	3		3	3							3		
CLO-6	3		3	3							3		

## Syllabus

- Unit I:** Introduction to R, data handling in R, data exploration, subsetting, handling factor variables in R, descriptive statistics, Fitting univariate, bivariate models to data, Data reduction/multivariate techniques (PCA, LDA, MDS, CA).
- Unit II:** Design of experiments in R: Effect of design on analysis of data, examples from CRD, RBD, Factorial design. Survey Sampling in R: Different sampling schemes, SRSWR, SRSWOR, stratified sampling, cluster sampling. Examples using "sampling" & "survey" packages. Point and Interval Estimation Tests for Continuous Data, Tests for Discrete Data.
- Unit III:** Basic Correlation and Regression, Multiple Regression with basic diagnostics, ANOVA. Generating random numbers in R, simulation based large sample approximation, simulation from stochastic processes, Bootstrapping.
- Unit IV:** Non-parametric Methods: Non-Parametric tests, Density estimation, Permutation tests. Nonlinear & non-parametric Regression: Common Transformations, Polynomial Regression, Box-Cox Transformations, Lowess.

### References / Reading Material :

1. Tanner, M. A., Tools for Statistical Inference, Springer-Verlag, 2011.
2. Snedcor, G. W. and Cochran, W. G., Statistical Methods, seventh edition, Iowa State University Press, 1982.
3. Conover, W. J., Practical Non-Parametric Statistics, third edition, John Wiley, NY, 2007.
4. Ripley, B.D., Stochastic Simulations, Wiley-Interscience, 2006.
5. Kennedy, W. J. and Gentle, J. E., Statistical Computing, Taylor & Francis, 1980.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Introduction to R, data handling in R, data exploration, subsetting, handling factor variables in R, descriptive statistics,	1
Week 2	I	Fitting univariate	2
Week 3	I	bivariate models to data	2
Week 4	I	Data reduction/multivariate techniques (PCA, LDA, MDS, CA)	2
Week 5	II	Design of experiments in R: Effect of design on analysis of data, examples from CRD, RBD, Factorial design. Survey Sampling in R: Different sampling schemes, SRSWR, SRSWOR, stratified sampling, cluster sampling. Examples using "sampling" & "survey" packages.	3
Week 6	II	Point and Interval Estimation	4
Week 7	II	Tests for Continuous Data	4
Week 8	II	Tests for Discrete Data.	4
Week 9	III	Basic Correlation and Regression, Data	4
Week 10	III	Multiple Regression with basic diagnostics, ANOVA.	4
Week 11	III	Generating random numbers in R, simulation based large sample approximation, simulation from stochastic processes	5
Week 12	III	Bootstrapping	5
Week 13	IV	Non-parametric Methods: Non-Parametric tests, Density estimation, Permutation tests.	6
Week 14	IV	Nonlinear & non-parametric Regression: Common Transformations, Polynomial Regression, Box-Cox Transformations, Lowess	6

## 2.37 Elements of Probability and Statistics, MA 304/ST 304/ST 405

**Course Code:** (MA 304/ST 304/ST 405): MSc Statistics-OR and IMSc in Mathematical Sciences

**Title of the Course:** Elements of Probability and Statistics

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main objectives of this course are to introduce students of M.Sc.(Statistics) to the concepts of Randomness, Probability and random variables.

### **Course Learning Outcomes (CLO's)**

After completion of this course successfully, the students will be able to

- CLO-1: Demonstrate understanding of uncertainty and actually identify it in nature and in different other phenomena and processes.
- CLO-2: Demonstrate ability to display the sample space for any random experiment.
- CLO-3: Explain what a random variable is and recognize different random variables in nature and phenomena and identify parameters.
- CLO-4: Determine probabilities of events and probability distributions of random variables.
- CLO-5: Determine whether or not moments exist of any given random variable and if so, to determine them.
- CLO-6: Apply tools like Probability Generating function and Moment generating functions - when they exist to study about distributions of some functions of random variables.
- CLO-7: Discuss and derive distributions of functions of random samples (Statistics) from different populations.
- CLO-8: Explain what statistics and their distributions(Sampling distributions) are and derive distributions of sample mean, sample variance and order statistics from some standard population distributions.

CLO-9: Demonstrate an understanding of different modes of convergence of sequences of random variables. Understand what some of the Laws of large numbers and the Central Limit Theorem say and see how they are used.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1		3			3		3	3			3		
CLO-2		3		3			3	3			3		
CLO-3	3	3					3	3			3		
CLO-4		3		3			3				3		
CLO-5	3	3					3				3		
CLO-6	3		3			3					3		
CLO-7		3		3			3				3		
CLO-8		3		3			3				3		
CLO-9		3		3	3		3				3		

## Syllabus

- Unit I:** Random experiments, sample spaces, sets, events,  $\sigma$ -algebras, probability. Elements of combinatorial analysis, classical definition and calculation of probability, independence of events, conditional probability, Bayes theorem. Random variables, distribution functions, Standard discrete and continuous.
- Unit II:** Expectations, raw and central moments, probability and moment generating functions, Compound, truncated and mixture distributions. Inequalities: Markov, Holder, Minkowski, Jensen and Liapunov inequalities, etc. Failure rate, their use in defining classes of distributions (e.g. aging classes), series & parallel systems. Functions of random variables and their distributions using Jacobian of transformation and other tools.
- Unit III:** Bivariate CDF (non-standard examples), bivariate moment generating function, Bivariate normal, multivariate normal, Joint, marginal and conditional PMFs. and PDFs. Conditional expectation, correlation, multiple and partial correlation. Linear and multiple regression. Sampling distributions. Non-central chi-square, t- and F- distributions and their properties. Distributions of quadratic forms under normality and related distribution theory. Fisher Cochran theorem (proof not given anymore), Examples of use of sampling distributions.

**Unit IV:** Order statistics, CI for p-th percentile (using order statistics), order statistics from standard distributions. Modes of convergence, Laws of large numbers, CLT.

**References / Reading Material :**

1. Dudewicz, E.J. and Mishra, S.N., Modern Mathematical Statistics, Wiley, Int'l Students' Edition, 1988.
2. Rohatgi, V.K., An Introduction to Probability Theory and Mathematical Statistics, Wiley Eastern, 1984.
3. Rao, C.R., Linear Statistical Inference and Its Applications, 2/e, Wiley Eastern, 1973.
4. Pitman, J., Probability, Narosa Publishing House, 1993.
5. Johnson, S. and Kotz, Distributions in Statistics, Vol. I, II and III, Houghton and Mifflin, 1972.
6. Cramer H., Mathematical Methods of Statistics, Princeton, 1946.

**Course Plan**

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Random experiments, sample spaces, sets, events, $\sigma$ -algebras, probability.	1
Week 2	I	Elements of combinatorial analysis, classical definition and calculation of probability	1
Week 3	I	Independence of events, conditional probability, Bayes theorem.	1
Week 4	I	Random variables, distribution functions, Standard discrete and continuous.	1,2,3
Week 5	II	Expectations, raw and central moments, probability and moment generating functions, Compound, truncated and mixture distributions.	1,2,3,4
Week 6	II	Inequalities: Markov, Holder, Minkowski, Jensen and Liapunov inequalities, etc.	5
Week 7	II	Failure rate, their use in defining classes of distributions (e.g. aging classes), series & parallel system.	5
Week 8	II	Functions of random variables and their distributions using Jacobian of transformation and other tools.	6
Week 9	III	Bivariate CDF (non-standard examples), bivariate moment generating function, Bivariate normal, multivariate normal, Joint, marginal and conditional p.m.f.s. and p.d.f.s.	7
Week 10	III	Conditional expectation, correlation, multiple and partial correlation. Linear and multiple regression.	8
Week 11	III	Sampling distributions. Non-central chi-square, t- and F- distributions and their properties.	8,9
Week 12	III	Distributions of quadratic forms under normality and related distribution theory. Fisher-Cochran theorem (proof not given anymore), Examples of use of sampling distributions	8,9
Week 13	IV	Order statistics, CI for p-th percentile (using order statistics), order statistics from standard distributions.	12
Week 14	IV	Modes of convergence, Laws of large numbers, CLT	13

## 2.38 Probability and Measure Theory, ST 351/ST 452

**Course Code:** (ST 351/ST 452):I.M.Sc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Probability and Measure Theory

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The aim of the course is to pay a special attention to applications of measure theory in the probability theory, understanding of Weak Law of Large Numbers, Strong Law of Large Numbers and the Central Limit Theorem with their applications.

**Prerequisite Course/Knowledge (if any):** Real-Analysis-I, Elements of Probability and Statistics

### **Course Learning Outcomes (CLO's)**

After completion of this course successfully, the students will be able to

- CLO-1: Apply important concepts from set -theory, well-ordered sets, and metric spaces.
- CLO-2: Explain the concepts like outer measure, pre- measure, Caratheodory, extension Theorem.
- CLO-3: Explain the concepts of Field, sigma Field and Measure of Borel-Sigma-Fields.
- CLO-4: Integrate non-negative functions and complex functions under measures.
- CLO-5: Explain different models of convergence, independence of events and functions of random variables with examples.
- CLO-6: Apply the concept of convergence of sequences of random variables in statistical analysis.
- CLO-7: Apply the concepts of weak and strong laws of large numbers and central limit theorem.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	3	3									3		
CLO-2	3	3									3		
CLO-3	3	3									3		
CLO-4		3									3		
CLO-5	3	3									3		
CLO-6	3	3									3		
CLO-7	3	3									3		

## Syllabus

- Unit I:** Concepts from set theory, orderings, cardinality, well ordered sets, axioms of choice, extended real number system, metric spaces, Bolzano-Weierstrass property, Heine Borel property, sigma-algebras, Borel-sigma-algebras, elementary family, measures, measurable space, measure space, properties of measure, outer measures, Caratheodory's Theorem, pre-measure
- Unit II:** Borel measures on the real line, distribution function, Lebesgue-Stieltjes measure, Lebesgue measure, measurable functions, characteristics function, integration of non-negative functions, Monotone convergence theorem, Fatou's Lemma, integration of complex functions, Dominated Convergence theorem.
- Unit III:** Riemann integral, models of convergence, Cauchy measure, product measures, Monotone Class lemma, Fubini Tonelli Theorem, signed measures, Radon-Nikodym theorem (statement only), probability, distribution function, random variables, independence, functions of random variables, sequence of random variables, modes of convergence.
- Unit IV:** Weak law of large numbers, Borel-Centelli Lemma, Kolmogorov's Inequality, Strong Law of large numbers, central limit theorem, construction of sample spaces.

## References / Reading Material :

1. Folland, G.B., Real Analysis: Modern Techniques and Their Applications, 2nd edition, Wiley Interscience Series of Texts, Monographs and Tracts, 1999.



2. Ash, R. B. and Doléans-Dade, C.A., Probability and Measure Theory, Second Edition, Academic Press, New York, 1999.
3. Billingsley, P., Probability and Measure, Anniversary Edition, John Wiley & Sons, 2012.
4. Dudley, R. M., Real Analysis and Probability, Wadsworth and Brooks/Cole, 1989.
5. Kingman, J. F. C. and Taylor, S. J., Introduction to Measure and Probability, Cambridge University Press, 1966.
6. Bhat, B.R., Modern Probability Theory, 3rd Edition, New Age International Publishers, 1999.
7. Capinski, M. and Zastawniah, Probability through problems, Springer, 2001.
8. Chung, K. L., A Course in Probability Theory, 2nd Edition, Academic Press, New York, 1974.
9. Feller, W., An Introduction to Probability Theory and its Applications, Vol. 1, 3rd Edition, John Wiley & Sons, 1968.
10. Parzen, E., Modern Probability Theory and its Application. Wiley Eastern Private Ltd., 1960.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	concepts from set theory, orderings, cardinality, well ordered sets, axioms of choice, .	1
Week 2	I	extended real number system, metric spaces, Bolzano Weierstrass property, Heine Borel property,	1
Week 3	I	Sigma-algebras, Borel-sigma-algebras, elementary family, measures, measurable space, measure space, properties of measure,	1,2,3
Week 4	I	outer measures, Caratheodory's Theorem, pre-measure.	2,3
Week 5	II	Borel measures on the real line, distribution function,	3,4
Week 6	II	Lebesgue-Stieltjes measure, Lebesgue measure, measurable functions,	5
Week 7	II	characteristics function, integration of non-negative functions, Monotone convergence theorem, Fatou's Lemma,	5
Week 8	II	integration of complex functions, Dominated Convergence theorem,	5
Week 9	III	Riemann integral, models of convergence, Cauchy measure,	5
Week 10	III	product measures, Monotone Class lemma, Fubini Tonelli Theorem	5
Week 11	III	signed measures, radon-Nikodym theorem,	5
Week 12	III	probability, distribution function, random variables, independence, functions of random variables, sequence of random variables, modes of convergence,	6,7
Week 13	IV	Weak of large numbers, Borel-Centelli Lemma, Kolmogorov's Inequality,	7,8
Week 14	IV	Strong Law of large numbers, central limit theorem, construction of sample spaces,	8

## 2.39 Linear Models, ST 353/ST 454

**Course Code:** (ST 353/ST 454): IMSc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Linear Models

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main purpose is to provide the theoretical foundations for the Linear Estimation Theory, Hypothesis testing, Analysis of Designed Experiments and Regression Analysis.

**Prerequisite Course/Knowledge (if any):** Linear Algebra and Matrix Theory

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

CLO-1: Apply advanced algebraic techniques in inference of linear models.

CLO-2: Apply different methods to estimate and test the relation between the independent and dependent variables.

CLO-3: Explain the concept and the method of generalized least squares.

CLO-4: Apply the method of generalized least squares for the treatment of any given set of data.

CLO-5: Test the general linear hypothesis, Confidence and Prediction intervals.

CLO-6: Explain Breaking up Sums of Squares and Types of sums of squares with examples.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	3	3	3	3			3	3			3		
CLO-2			3	3			3		3		3		
CLO-3							3	3					
CLO-4		3		3	3		3				3		
CLO-5	3	3	3	3			3				3		
CLO-6	3	3						3			3		

## Syllabus

- Unit I:** Review of: linear algebra, orthogonal projection, Random vectors and matrices, Multivariate Normal distribution (MVN) based on affine transformation, properties of MVN, sampling distributions, linear and quadratic forms.
- Unit II:** The full rank linear model — Estimation and Inference. The linear Model, Least Square Estimation, Gauss- Markov Theorem, Maximum Likelihood Estimation, Estimation of  $\sigma^2$ , MVUE.
- Unit III:** Generalized Least Squares. Over-fitting, under-fitting. Model in Centered form, Estimated Coefficient of Determination. Testing a Subset of  $\beta$ . Overall Regression Test. Testing of general linear hypothesis, Confidence and Prediction Intervals. Analysis of Variance Models: The non-full rank case (e.g., ANOVA models) — Estimability and testability, parameterizations, and constraints.
- Unit IV:** Breaking up Sums of Squares, Types of sums of squares.

### References / Reading Material :

1. Rao, C. R., Linear Statistical Inference and Its Applications, second edition, Wiley Eastern, 1973.
2. Rao, A. R. and Bhimasankaram, P., Linear Algebra, second edition, TRIMHindustan Book Agency, 2000.
3. Sengupta, D. and Jammalamadaka Rao, S., Linear Models-An Integrated Approach, World Scientific, 2003.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Statistical Model for a phenomenon, motivation through some examples. Review of: linear algebra, orthogonal projection.	1
Week 2	I	Orthogonal projection Random vectors and matrices	1
Week 3	I	Multivariate Normal distribution (MVN) based on affine transformation, properties of MVN,	1
Week 4	I	Sampling distributions, linear and quadratic forms.	1
Week 5	II	The full rank linear model — Estimation and Inference.	1,2
Week 6	II	The linear Model, Least Square Estimation	1,2
Week 7	II	Gauss- Markov Theorem	1,2
Week 8	II	Maximum Likelihood Estimation, Estimation of $\sigma^2$ . MVUE	1,2
Week 9	III	Generalized Least Squares. Overfitting, under-fitting. Model in Centered form, Estimated Coefficient of Determination	2,3
Week 10	III	Testing a Subset of $\beta$ . Overall Regression Test. Testing of general linear hypothesis, Confidence and Prediction Intervals.	2,3
Week 11	III	Analysis of Variance Models: The non-full rank case	2,3
Week 12	III	Estimability and testability Parameterizations, and constraint	2,3
Week 13	IV	Breaking up Sums of Squares	2,3
Week 14	IV	Types of sums of squares.	2,3

## 2.40 Theory of Sampling, ST 354/ST 455

**Course Code:** (ST 354/ST 455):I.M.Sc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Theory of Sampling

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main objective is to provide the knowledge of concept of sample and population in statistics and also the various sampling schemes and estimation of population parameters and their respective standard errors.

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1 Explain and suggest where and when sample studies and surveys are needed and the pros and cons of a complete enumeration study.
- CLO-2 Suggest what parameters on the population and what the population frame should be to experts from other fields in their research.
- CLO-3 Explain the various sampling schemes and designs, SRSWOR, SRSWR, Stratified, Systematic, Cluster sampling, Sampling with varying probabilities and how estimators and sample sizes are to be determined.
- CLO-4 Be clear about sampling errors and Non-sampling errors and a student who has studied this course will be able to explain these and convince anyone for the need of exploring sampling designs and plans that will minimize sampling errors.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1		3			3		3				3		
CLO-2		3	3		3	3	3						
CLO-3	3	3	3	3		3		3			3		
CLO-4		3			3	3	3				3		

## Syllabus

The role of sampling theory, simple random sampling for mean, proportions and percentages, estimation of sample size.

Stratified sampling for mean and proportions, optimum allocation, relative precision of stratified random and simple random sampling, effect of deviations from the optimum allocation, effect of errors in the stratum sizes.

Systematic sampling for linear and circular cases, variance of estimated mean, comparison of systematic sampling with simple random sampling and stratified sampling for population with linear trend.

Procedure of selecting a sample with varying probabilities, estimation of the population mean and variance of the estimated mean, sampling with varying probabilities and without replacement, ordered estimates, the Horvitz-Thompson Estimator, Some IPPS sampling without replacement procedures, Rao Hartley and Cochran's procedure.

Ratio Estimator, bias and mean squared error of the ratio estimator and its approximation, ratio estimates in stratified sampling, comparison of the ratio estimate and the mean per unit.

Regression estimator, bias and mean squared error of the regression estimator, efficiency of the regression estimator, regression estimates in stratified sampling,

### References / Reading Material :

1. Cochran, W. C., Sampling Techniques, second edition, third edition, Wiley Eastern, 1977.
2. Sukhatme, P. V.; Sukhatme, B. V.; Sukhatme, S. and Asok, C., Sampling Theory of Surveys with Applications, Indian Society of Agricultural Statistics, 1954.
3. Raj, Des, Sampling Theory, Tata McGraw-Hill, 1968.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Need for sample studies and surveys, drawbacks of complete enumeration, Sampling error and Non-sampling error. How such studies are initiated.	1
Week 2	I	Identification of: 'Population frame', variable of interest, population members (or units). Sample as an outcome of a random experiment and sample statistics (estimators) as random variables.	1
Week 3	I	SRSWR and SRSWOR. Estimating proportions and percentages.. Estimation of sample size.	1, 3
Week 4	I	Using SRSWOR to estimate population size-when population units are Labeled or unlabeled.	1, 3
Week 5	II	Stratification of population units: Criteria Estimators ??? for population Mean, Total and Proportions, their variances Sample sizes allocation: Proportional allocation, Optimum Allocation, Comparisons of variances of the unbiased estimators under the different allocations.	2, 3
Week 6	II	Relative Precisions of estimators for Stratified random and Simple Random Sampling.	2, 3
Week 7	II	Systematic Sampling: unbiased estimator for population mean and total. effect of within correlation on the efficiency, systematic sample in the presence of a linear trend.	2, 3
Week 8	II	comparison between Systematic Sampling and simple random sampling comparison in the presence of stratification	2, 3
Week 9	III	Using information on auxiliary variables: Ratio estimators. Estimators for population mean and total based on estimating the ratio of means and mean of ratios, their bias and mean squared errors	2, 3
Week 10	III	Regression estimator: bias and mean squared error of the regression estimator, efficiency of the regression estimator.	3, 4
Week 11	III	Cluster Sampling:Single stage: Equal and unequal cluster sizes: Unbiased estimator for population mean per cluster and population mean across all clusters. Their variances.	3,4
Week 12	III	Two-stage cluster sampling.	3, 4
Week 13	IV	Sample Selection with varying probabilities: Drawing a sample in which population units may have different probabilities of being selected: PPS: Lahiri's scheme, Midzuono's scheme, Rao Hartley and Cochran's procedures. samples with and without replacement. unbiased estimators and determination of their variances.	3, 4
Week 14	IV	Desraj and Horvitz-Thompson (HT) estimators for population mean and population total. Resolution of the problem of negative mean squared error of the HT estimator.	3, 4

## 2.41 Analyzing Large Data with R, ST 355/ ST 457

**Course Code:** (ST 355/ST 457): IMSc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Analyzing Large Data with R

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** In this course student will learn how to write scalable and efficient R code and ways to visualize it too.

**Pre-requisite Course/Knowledge (if any):** Statistical Methods

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Explain R data objects and how large data can be handled for analysis.
- CLO-2: Enter and manipulate large data in R in a way that makes sense.
- CLO-3: Use R to model large data.
- CLO-4: Apply the principles of R to combine additional data resources in R.
- CLO-5: Discuss the applications of R in different disciplines of knowledge and in different contexts.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	3		3								3		
CLO-2	3		3								3		
CLO-3	3		3								3		
CLO-4	3		3								3		
CLO-5	3		3								3		



## Syllabus

- Unit I:** Large Data: Basic properties of Large data, source of Large data, types of data, data cleaning, handling and processing, methodological challenges and problems, applications. Pre-processing: Reading, selecting data, filtering data, handling missing values. Visualization: issues with large-Scale data visualization, softwares, examples.
- Unit II:** Data transformation: Smoothing, manipulating, power transformations, Min-Max normalization, Z-transformation, Box-Cox transformation, Lowess-method. Data Exploration: Data summarization, clustering, Dimensionality Reduction.
- Unit III:** Modeling: Linear Models, multi-collinearity in large data, PCA-based regression, non-linear regression, categorical response modeling, classification and regression trees.
- Unit IV:** Other big data: Data integration, multi source data, multi-level data, linked data, meta data.

### References / Reading Material :

1. Larose, D.T., Larose, C.D., Data Mining and Predictive Analytics, Wiley, 2015.
2. Sandy Ryza, Uri Laseron, Sean Owen and Josh Wills, Advanced Analytics with Spark, O' Reilly, 2015.
3. Prabhanjan, N.T., Suresh, R., Manjunath, B.G., A Course in Statistics with R, Wiley, 2016.
4. Simon Walkowiak, Big Data Analytics with R, Packet, 2016.

### Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Large Data: Basic properties of Large data, source of Large data, types of data	1
Week 2	I	Data cleaning, handling and processing, methodological challenges and problems, applications.	1,2
Week 3	I	Exponential families, Pitman families, Curved exponential Pre-processing: Reading, selecting data, filtering data, handling missing values.	1,2
Week 4	I	Visualization: issues with large-Scale data visualization, softwares, examples.	1,2
Week 5	II	Data transformation: Smoothing, manipulating	2
Week 6	II	Power transformations, Min-Max normalization,	2
Week 7	II	Z-transformation, Box-Cox-transformation Lowess-method	2
Week 8	II	Data Exploration: Data summarization, clustering, Dimensionality Reduction.	2
Week 9	III	Properties of maximum likelihood estimators and its asymptotic Modeling: Linear Models,	3
Week 10	III	Multi-collinearity in large data, PCA-based regression,	3
Week 11	III	Non-linear regression	3
Week 12	III	Categorical response modeling, classification and regression trees.	3
Week 13	IV	Other big data: Data integration, multi source data	4
Week 14	IV	Multi-level data, linked data, meta data.	4

## 2.42 Theory of Inference - I, ST 352/ST 453

**Course Code:** (ST 352/ST 453): I.M.Sc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Theory of Inference – I

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** To make students aware of estimation of point, as well as, interval.

**Pre-requisite Course/Knowledge (if any):** Elements of Probability and Statistics, basic mathematics, calculus

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

CLO-1: Discuss the concept of Point estimation and its properties.

CLO-2: Apply sufficiency, ancillary and likelihood principles.

CLO-3: Understand Fisher Information, Lower bounds to variance of estimators, MVUE.

CLO-4: Discuss the concept of interval estimation and its properties.

CLO-5: Determine shortest expected length confidence interval.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	3							3			3		
CLO-2	3							3			3		
CLO-3	3							3			3		
CLO-4	3							3			3		
CLO-5	3							3			3		

## Syllabus

### PROBLEM OF POINT ESTIMATION:

Properties of point estimation: Unbiasedness, closeness, mean squared error, consistency and BAN. Methods of finding estimators: Method of moments, maximum likelihood, other methods; sufficient, complete and ancillary statistics. unbiased estimation: Lower bound for the variance of an estimator; location and scale invariant estimators; optimum properties of maximum likelihood estimation.

### PARAMETRIC INTERVAL ESTIMATION:

Definition of confidence interval, pivotal quantity. Sampling from the normal distribution : Confidence interval for the mean, confidence interval for the variance, simultaneous confidence region for the mean and variance, confidence interval for difference in means. Methods of finding confidence intervals: Pivotal quantity method, statistical method, large sample confidence intervals.

### References / Reading Material :

1. Rohatgi, V. K., An Introduction to Probability and Statistics, Wiley Eastern, 1985.
2. Casella, G. and Berger, R. L., Statistical Inference, second edition, Academic Internet Pub., 2002.
3. Mood, A. M.; Graybill, F. A. and Boes, D. C., Introduction to the Theory of Statistics, third edition, McGraw-Hill, 1974.
4. Lehmann, E. L., Theory of Point Estimation, John Wiley, NY, 1983.
5. Lehmann, E.L., Testing Statistical Hypotheses, John Wiley & Sons, 1986.
6. Ferguson, T.S., Mathematical Statistics, Academic Press, 1967.
7. Kale, B.K, A First Course on Parametric Inference, Narosa Publishing House, 1999.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Introduction to point estimation	1
Week 2	I	Minimal sufficiency and ancillarity.	2
Week 3	I	Exponential families and Pitman families.	2
Week 4	I	Invariance property of Sufficiency under one-one transformations of sample and parameter spaces.	2
Week 5	II	Unbiased estimators and its properties	3
Week 6 & 7	II	Rao-Blackwell and Lehmann-Scheffe Theorems. Lower bounds to variance of estimators, necessary and sufficient conditions for MVUE.	3
Week 8	III	Basu Theorem and Methods of finding estimators: Method of Moments, Maximum likelihood, Least Square. family.	3
Week 9	III	Properties of maximum likelihood estimators and its asymptotic Extension of these results to Pitman family when only upper or lower end depends on the parameters and to distributions with MLR property.	3
Week 10	III	BAN and CAN estimators	3
Week 11 & 12	III	Interval estimation, confidence level, construction of shortest expected length confidence interval.	4
Week 13	IV	Uniformly most accurate one-sided confidence Interval.	5
Week 14	IV	Shortest expected length confidence interval and Asymptotic confidence interval.	5

## 2.43 Multivariate Analysis, IM 401/ST 501

**Course Code:** (IM 401/ST 501):IMSc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Multivariate Analysis

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main objective is to learn analyzing multivariate data, to increase familiarity with the handling of multivariate data and applications of multivariate techniques to data through software.

**Pre-requisite Course/Knowledge (if any):** Linear Algebra and Matrix Theory (ST-302)

### **Course Learning Outcomes (CLO's)**

After completion of this course successfully, the students will be able to

- CLO-1 Explain and discuss the different multivariate distributions, their properties and characterizations.
- CLO-2 Explain the inadequacy of euclidean distance between two data points of a multivariate random vector explain how measures based on the dispersion matrix is more suitable and test for multivariate normality, distribution as being from the multivariate normal population.
- CLO-3 Estimate parameters of the MVN distribution and confidence regions. Carry out hypothesis tests for the multivariate normal distribution - on both the mean vector and the dispersion matrix.
- CLO-4 Explain multivariate techniques like PCA, Factor Analysis, Discriminant Analysis and use them on Multivariate data for analysis.
- CLO-5 Explain different ways of classification of variables and observations - hierarchical, clustering.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	3	3						3			3		
CLO-2		3		3			3	3			3		
CLO-3	3	3					3	3	3		3		
CLO-4			3	3	3		3		3		3		
CLO-5		3	3		3	3					3		

## Syllabus

- Unit I:** Concept of random vector and random matrix. Multivariate distribution function and marginal and conditional distribution. Bivariate distributions- Binomial, Poisson, Exponential distributions, Multinomial Dirichlet distributions; conditional, marginal distributions. Multivariate Normal Distribution (MVN), singular and non-singular MVN, properties of MVN, marginal distributions, conditional distributions, linear transformations, characteristics function, estimation of MVN parameters, sampling distributions of MLEs, hypothesis testing for MVN mean, distribution of sample mean vector and its independence.
- Unit II:** Multiple linear equations, Multiple correlation, partial correlation in multiple setup. Hotelling's  $T^2$  and its applications. Hotelling's  $T^2$  statistic as a generalization of square of Student's statistic. Distance between two populations, Mahalanobis  $D^2$  statistic and its relation with Hotelling's  $T^2$  statistic. Wishart distribution, its properties, usage in confidence region construction.
- Unit III:** Multivariate regression, MANOVA, MANCOVA. Multivariate techniques- Principle component analysis, Discriminant analysis, Multidimensional scaling, Factor Analysis, Canonical correlation.
- Unit IV:** Classification problem, Cluster Analysis.

## References / Reading Material :

1. Johnson, R.A. and Wichern. D.W., Applied multivariate Analysis. 5thAd.Prentice –Hall, 2002.
2. Mardia, K.V., Kent, J.T., Bibby, J.M., Multivariate analysis, Academic Press, 1979.

3. Anderson, T. W., Introduction to Multivariate Analysis, John Wiley, 1984.
4. Kshirsagar A. M. : Multivariate Analysis. Maral-Dekker, 1979.
5. Kotz, S., Balakrishnan N. and Johnson N. L., Continuous Multivariate Distributions, Volume 1, Models and Applications, John Wiley & Sons, 2000.

### **Course Plan**

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Concept of random vector and random matrix. Multivariate distribution function and marginal and conditional distribution.	1
Week 2	I	Bivariate distributions- Binomial, Poisson, Exponential distributions, Multinomial Dirichlet distributions; conditional, marginal distributions.	1,2
Week 3	I	Multivariate Normal Distribution (MVN), singular and non-singular MVN, properties of MVN, marginal distributions, conditional distributions, linear transformations, characteristics function,	1,2
Week 4	I	Estimation of MVN parameters, sampling distributions of MLEs, hypothesis testing for MVN mean, Distribution of sample mean vector and its independence..	1,2,3
Week 5	II	Multiple linear equations, Multiple correlation, partial correlation in multiple setup.	1,2,3
Week 6	II	Hotelling's $T^2$ and its applications. Hotelling's $T^2$ statistic as a generalization of square of Student's statistic.	1,2,3
Week 7	II	Distance between two populations, Mahalanobis $D^2$ statistic and its relation with Hotelling's $T^2$ statistic	2,3,4
Week 8	II	Wishart distribution, its properties, usage in confidence region construction	3,4
Week 9	III	Multivariate regression, MANOVA, MANCOVA	3,4
Week 10	III	Multivariate techniques- Principle component analysis, PCA based regression	4,5,6
Week 11	III	Non-linear Discriminant analysis, Multidimensional scaling	4,5,6
Week 12	III	Factor Analysis, Canonical correlation.	4,5,6
Week 13	IV	Classification problem	4,5,6
Week 14	IV	Multi-level data, linked data, meta Cluster Analysis.	4,5,6



## 2.44 Design and Analysis of Experiments, IM 403/ ST 503

**Course Code:** (IM 403/ST 503): I.M.Sc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Design and Analysis of Experiments

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main objective is to provide the theoretical foundations for design and analysis of experiments.

**Pre-requisite Course/Knowledge (if any):** Linear Algebra and Matrix Theory (ST-302)

### **Course Learning Outcomes (CLO's)**

After completion of this course successfully, the students will be able to

- CLO-1: Apply ANOVA for 1 and 2-way classification, for (a) fixed effect models with equal, unequal and proportional number of observations per cell, and (b) Random and Mixed effect models with  $m(> 1)$  observations per cell.
- CLO-2: Discuss and use incomplete block designs, discuss the concepts of orthogonality, connectedness and balance.
- CLO-3: Discuss the concepts of finite fields and finite geometries and apply them in construction of Latin Squares, balanced incomplete block designs, confounded factorial experiments.
- CLO-4: Identify the effects of different factors and their interactions and analyze factorial experiments.
- CLO-5: Apply appropriate factorial designs and perform their analysis, practical utility of Split-plot designs, understand the effects of independence or dependence of different factors under study.
- CLO-6: Discuss background and utility of response surface designs.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO1	3	3	3				3	3			3		
CLO2	3	3	3				3	3			3		
CLO3	3												
CLO4		3	3	3			3	3			3		
CLO5		3	3	3			3	3			3		
CLO6		3	3	3			3	3			3		

## Syllabus

Review of linear estimation and basic designs. Elimination of heterogeneity in two directions. ANOVA: Fixed effect models (2-way classification with equal, unequal and proportional number of observations per cell), Random and Mixed effect models (2-way classification with  $m(> 1)$  observations per cell).

Block Designs: Connectedness, Orthogonality, Balance and Efficiency; Resolvable designs. Properties of BIB designs, Designs derived from BIB designs. Intrablock analysis of BIB, Recovery of inter-block information in BIB designs; Row column and Youden Square designs, Missing plot technique. Elementary ideas of Lattice and PBIB designs.

Finite fields. Finite Geometries- Projective geometry and Euclidean geometry. Construction of complete set of mutually orthogonal latin squares (MOLS). Construction of BIBD using finite Abelian groups, MOLS, finite geometry and method of differences.

Factorial designs: Analysis, Confounding and balancing in Symmetric Factorials. Response Surface Designs.

## References / Reading Material :

1. Chakrabarti, M.C., Mathematics of Design and Analysis of Experiments, Asia Publishing House, Bombay, 1962.
2. Mardia, K.V., Kent, J.T., Bibby, J.M., Multivariate analysis, Academic Press, 1979.
3. Dey, A., Theory of Block Designs, John Wiley & Sons, 1986.
4. Montgomery, D. C., Design and Analysis of Experiments, 6th ed., John Wiley & Sons, 2005.

5. Raghavarao, D. and Padgett, L. V., Block Designs: Analysis, Combinatorics, and Applications, World Scientific, 2005.
6. Raghavarao, D., Construction and Combinatorial Problems in Design of Experiments, John Wiley & Sons, 1970.

### Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Review of linear estimation and basic designs,	1
Week 2	I	Elimination of heterogeneity in two directions.	1
Week 3	I	ANOVA, Fixed effect models (2-way classification with equal, unequal and proportional number of observations per cell).	1
Week 4	I	ANOVA, Random and Mixed effect models (2-way classification with $m (> 1)$ observations per cell).	1
Week 5	II	Incomplete Block Designs. Concepts of Connectedness, Orthogonality and Balancedness.	2
Week 6	II	Incomplete Block Designs. Concepts of Connectedness, Orthogonality and Balancedness.	2
Week 7	II	Intrablock analysis of General Incomplete Block design. BIBD with and without recovery of interblock information.	2
Week 8	II	Intrablock analysis of General Incomplete Block design. BIBD with and without recovery of interblock information.	2
Week 9	III	Finite fields	3
Week 10	III	Finite Geometries- Projective geometry and Euclidean geometry	3
Week 11	III	Construction of complete set of mutually orthogonal latin squares. Construction of BIBD using finite Abelian groups and MOLS.	3
Week 12	III	Construction of BIBD using finite geometry and method of differences.	3
Week 13	IV	Symmetrical factorial experiments ( $sm$ , where $s$ is a prime or a prime power), Confounding in $sm$ factorial experiments through Pencils. $sk-p$ fractional factorial where $s$ is a prime or a prime power. Split-plot experiments.	4,5
Week 14	IV	Response Surface Designs.	6

## 2.45 Theory of Inference - II, IM 402/ST 502

**Course Code:** (IM 402/ST 502): IMSc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Theory of Inference – II

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** To make aware the students of parametric, non-parametric and sequential estimation and testing (simple, as well as, composite hypotheses) procedures.

**Pre-requisite Course/Knowledge (if any):** Theory of Inference–I

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Apply various parametric, non-parametric and sequential estimation techniques and testing procedures to deal with real life problems.
- CLO-2: Discuss the concept of consistency, CAN estimator, MLE.
- CLO-3: Discuss UMPU tests, SPRT, OC and ASN.
- CLO-4: Discuss the difference between UMP and UMP unbiased test
- CLO-5: Apply non-parametric methods, U-statistics

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO1	3							3			3		
CLO2	3							3			3		
CLO3	3							3			3		
CLO4	3							3			3		
CLO5	3							3			3		

## Syllabus

- Unit I:** Consistency and asymptotic relative efficiency of estimators, Consistent asymptotic normal (CAN) estimator, Method of maximum likelihood, CAN estimator for one parameter Cramer family, Cramer-Huzurbazar theorem, Solutions of likelihood equations, method of scoring, Fisher lower bound to asymptotic variance, MLE in Pitman family and double exponential distribution, MLE in censored and truncated distributions.
- Unit II:** Similar tests, Neyman structure, UMPU tests for composite hypotheses, Invariance tests and UMP invariant tests, Likelihood ratio test, Asymptotic distribution of LRT statistic, Consistency of large sample test, Asymptotic power of large sample test.
- Unit III:** Sequential tests-SPRT and its properties, Wald's fundamental identity, OC and ASN functions. Sequential estimation.
- Unit IV:** Non- parametric methods-estimation and confidence interval, U-statistics and their asymptotic properties, UMVU estimator, nonparametric tests-single sample location, location-cum-symmetry, randomness and goodness of fit problems; Rank order statistics, Linear rank statistics, Asymptotic relative efficiency.

### References / Reading Material :

1. Rohatgi, V. K., An Introduction to Probability and Statistics, Wiley Eastern, 1985.
2. Casella, G. and Berger, R. L., Statistical Inference, second edition, Academic Internet Pub., 2002.
3. Mood, A. M. , Graybill, F.A., and Boes, D.C., Introduction to the Theory of Statistics, third edition, McGraw-Hill, 1974.
4. Lehmann, E. L., Theory of Point Estimation, John Wiley, NY, 1983.
5. Gibbons, J. D. and Chakaraborti, Nonparametric Statistical Inference, fourth edition, Marcel Dekker, 2003.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Consistency and asymptotic relative efficiency of estimators. Consistent asymptotic normal (CAN) estimator.	1
Week 2	I	Method of maximum likelihood, CAN estimator for one parameter Cramer family.	1
Week 3	I	Cramer-Huzurbazar theorem.	1
Week 4	I	Solutions of likelihood equations, method of scoring.	1
Week 5	I	Fisher lower bound to asymptotic variance. MLE in Pitman family and double exponential distribution, MLE in censored and truncated distributions.	1
Week 6	II	Similar tests, Neyman structure, UMPU tests for composite hypotheses.	2
Week 7	II	Invariance tests and UMP invariant tests.	2
Week 8	II	Likelihood ratio test,	2
Week 9	II	Asymptotic distribution of LRT statistic, Consistency of large sample test, Asymptotic power of large sample test	2
Week 10	III	Sequential tests-SPRT and its properties.	3
Week 11	III	Wald's fundamental identity, OC and ASN functions. Sequential estimation.	3
Week 12	IV	Non- parametric methods-estimation and confidence interval, U-statistics and their asymptotic properties.	4
Week 13	IV	UMVU estimator, nonparametric tests-single sample location, location-cum symmetry, randomness and goodness of fit problems; Rank order statistics.	4
Week 14	IV	Linear rank statistics, Asymptotic relative efficiency.	4

## 2.46 Regression Theory and Analysis, IM 404/ST 504

**Course Code:** (IM 404/ST 504): IMSc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Regression Theory and Analysis

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main purpose is to provide the theoretical foundations for Regression Analysis and enable in-depth practical implementation for quantitative analysis.

**Pre-requisite Course/Knowledge (if any):** Linear Models (ST-454)

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Discuss the different types of regression and when these are applicable
- CLO-2: Visualize simple linear regression model fits.
- CLO-3: Discuss the role of residuals
- CLO-4: Discuss the concept of multiple linear regression models and how these can be constructed
- CLO-5: Discuss and be able to apply different regression diagnostics
- CLO-6: Discuss and be able to apply alternative regression models.
- CLO-7: Apply the methods in a software and decipher output from the same software

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO1	3		3	3							3		
CLO2	3		3	3							3		
CLO3	3		3	3							3		
CLO4	3		3	3							3		
CLO5	3		3	3							3		
CLO6	3		3	3							3		
CLO7	3		3	3							3		

## Syllabus

- Unit I:** Simple Linear Regression Model, Least-Squares Estimation, Hypothesis Testing on the Slope and Intercept, Interval Estimation, Prediction of New Observations, Coefficient of Determination, Regression through the Origin, Maximum Likelihood estimation of parameters, case with Regressor  $x$  as Random. Multiple Regression Models, Estimation, Hypothesis Testing, Confidence Intervals, Prediction of New Observations, Hidden Extrapolation in Multiple Regression, Standardized Regression Coefficients, Multi-collinearity
- Unit II:** Model Adequacy Checking -Residual Analysis, PRESS Statistic, Detection and Treatment of Outliers, Lack of Fit of the Regression Model. Transformations and Weighting to Correct Model Inadequacies, Variance-Stabilizing Transformations, Transformations to Linearize the Model, Analytical Methods for Selecting a Transformation, Generalized and Weighted Least Squares, Regression Models with Random Effect. Diagnostics for Leverage and Influence-Influential Observations, Leverage, Cook's D, DFFITS and DFBETAS, A Measure of Model Performance, Groups of Influential Observations, Treatment of Influential Observations. Polynomial Regression Models- Models in One Variable, Non-parametric Regression, Models in Two or More Variables, Orthogonal Polynomials
- Unit III:** Indicator Variables-Concept, Use of Indicator Variables, Regression Approach to Analysis of Variance. Multicollinearity-Sources, Effects, Diagnostics, Methods for Dealing with Multicollinearity. Variable Selection and Model Building- Techniques for Variable Selection, Strategy for Variable Selection and Model Building. Validation of Regression Models-Techniques
- Unit IV:** Nonlinear Regression- Models, Origins of Nonlinear Models, Nonlinear Least Squares. Other Topics-Robust Regression, Measurement Errors, Calibration Problem, Bootstrapping in Regression

### References / Reading Material :

1. Montgomery, D. C.; Peck, E. A. and Vining, G. G., Introduction to Linear Regression Analysis, third edition, John Wiley, 2003.
2. Draper, N. R. and Smith, H., Applied Regression Analysis, third edition, John Wiley & Sons, NY, 2005.



3. McCullagh, P. and Nelder, J. A., Generalized Linear Models, second edition, CRC Books, London, 2000.
4. Seber, G. E. F. and Wild, C. J., Nonlinear Regression, John Wiley, NY, 1989.
5. Neter, J.; Kutner, M.; Wasserman, W. and Nachtsheim, C., Applied Linear Statistical Models., fourth edition, McGraw-Hill/Irwin, 1996.

### Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Simple Linear Regression Model, Least-Squares Estimation, Hypothesis Testing on the Slope and Intercept, Interval Estimation, Prediction of New Observations,	1,2,7
Week 2	I	Coefficient of Determination, Regression through the Origin, Maximum Likelihood estimation of parameters, case with Regressor x as Random.	2,7
Week 3	I	Multiple Regression Models, Estimation, Hypothesis Testing, Confidence Intervals,.	2,7
Week 4	I	Prediction of New Observations, Hidden Extrapolation in Multiple Regression, Standardized Regression Coefficients, Multi-collinearity	3,7
Week 5	II	Model Adequacy Checking -Residual Analysis, PRESS Statistic, Detection and Treatment of Outliers, Lack of Fit of the Regression Model	3,7
Week 6	II	Transformations and Weighting to Correct Model Inadequacies, Variance-Stabilizing Transformations, Transformations to Linearize the Model, Analytical Methods for Selecting a Transformation, Generalized and Weighted Least Squares, Regression Models with Random Effect	4,7
Week 7	II	Diagnostics for Leverage and Influence-Influential Observations, Leverage, Cook's D, DFFITS and DFBETAS, A Measure of Model Performance, Groups of Influential Observations, Treatment of Influential Observations	5,7
Week 8	II	Polynomial Regression Models- Models in One Variable, Nonparametric Regression, Models in Two or More Variables, Orthogonal Polynomials	6,7
Week 9	III	Indicator Variables-Concept, Use of Indicator Variables, Regression Approach to Analysis of Variance	6,7
Week 10	III	Finite Geometries- Projective geometry and Euclidean Multicollinearity-Sources, Effects, Diagnostics, Methods for Dealing with Multicollinearity	5,6,7
Week 11	III	Variable Selection and Model Building- Techniques for Variable Selection, Strategy for Variable Selection and Model Building	5,6,7
Week 12	III	Validation of Regression Models-Techniques	7
Week 13	IV	Nonlinear Regression- Models, Origins of Nonlinear Models, Nonlinear Least Squares	6,7
Week 14	IV	Other Topics-Robust Regression, Measurement Errors, Calibration Problem, Bootstrapping in Regression	6,7

## 2.47 Stochastic Processes, IM 405/ST 505

**Course Code:** (IM 405/ST 505): I.M.Sc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Stochastic Processes

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main objective of the paper is to provide theoretical foundations of Stochastic Processes and to introduce different Stochastic/Random Processes and their applications.

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Explain what a stochastic process and sample is. Discuss the principles and objectives of model building based on Markov chains, Poisson processes and Brownian motion.
- CLO-2 Discuss the principles and objectives of model building based on Markov chains, Poisson processes, Markov processes, Brownian motion.
- CLO-3: Determine transition matrices for Markov dependent behavior and summarize process information
- CLO-4: Apply the various birth and death models to explain population growth, epidemics and queues.
- CLO-5: Discuss the notions of long-time behavior including transience, recurrence, and equilibrium in applied situations such as branching processes and random walk.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO1	3	3	3				3	3			3		
CLO2		3	3	3			3				3		
CLO3	3	3					3	3					
CLO4	3	3				3	3	3			3		
CLO5		3	3	3			3	3					

## Syllabus

Introduction, Poisson process - Introduction, homogeneous and the non-homogeneous Poisson processes, renewal process, key renewal theorem and some applications. Markov Chains : classification of states, existence of stationary distribution, expected time between successive visits to a state - positive recurrent states, Random walk, Markov Processes, Continuous time discrete state Markov processes, embedded Markov chains and semi-Markov processes, various birth and death processes-Yule Process, the various Markovian queues and their steady state distribution when they exist.

Martingale processes : martingale theorem and some applications, Doob's inequality.

Brownian motion process as a limit of the random walk, its properties, Brownian bridge.

### References / Reading Material:

1. Ross, S., Stochastic processes, second edition, John Wiley, 1996.
2. Goswami, A. and Rao, B. V., A Course in Applied Stochastic Processes, TRIM Hindustan Book Agency, 2006.

### Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Introduction to Stochastic Processes, Some examples	1
Week 2	I	The Poisson Process-definition, study of its properties, Non-Homogeneous Poisson Process.	1
Week 3	I	Renewal Processes - Renewal theorem, stopping time. Key Renewal theorem, its applications..	1
Week 4	I	Alternating Renewal Process, Age and Residual life Delayed Renewal Processes - Working out examples.	1
Week 5	II	Homogeneous Markov chain-definition, examples, Transition probability matrices Chapman-Kolmogorov equation. Recurrent and Transient states, Classification of states into communicating classes, periods of states.	2
Week 6	II	Criteria to determine if a state is recurrent Determining whether an irreducible Markov chain is reducible or not Expected number of transitions to return to a state Positive and Null recurrence.	2
Week 7	II	Stationary and Long run distributions of a Markov chain - existence or non-existence, Determining them when they exist, determining whether they exist criterion for an irreducible Markov chain to be positive recurrent and then computing the mean times to return	2
Week 8	II	Branching process and determining probability of extinction. Continuous time discrete state Markov processes-definition $\nu_i$ and $q_{ij}$ for Markov processes.	1,3
Week 9	III	Kolmogorov Forward and backward differential equations - derivation and applying for some examples. Birth and Death processes, Several exams to be discussed.	3
Week 10	III	Semi-Markov processes, Steady state distributions of such processes based on the TPM of the embedded Markov chain. Existence of these for the $M-M-1$ and $M-M-c$ queues.	3
Week 11	III	Random Walk, Random times and Stopping times.	3
Week 12	III	Martingale Process - Denition, examples. Stopping time for a martingale Process Stopped process corresponding to a Martingale process. Wald's equation when expected value of stopping time exists. Example - classical one of determining expected number of independent trials till a certain sequence appears.	3
Week 13	IV	Doob's inequality and its applications,Sub-Martingales and Super Martingales. Brownian Motion Process - As a limit of the simple symmetric random walk	3,4
Week 14	IV	Standard Brownian Motion Process and Brownian Motion with drift, Brownian Bridge	4

## 2.48 Generalized Linear Models, IM 451/ST 552

**Course Code:** (IM 451/ST 552): IMSc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Generalized Linear Models

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main objective of this course is to provide students the ability to learn and use generalized linear models for normal and non-normal responses.

**Pre-requisite Course/Knowledge (if any):** Theory of Inference – I (ST-453), Theory of Inference – II (ST-502)

### **Course Learning Outcomes (CLO's)**

After completion of this course successfully, the students will be able to

- CLO-1: Discuss linear and Non-linear models, apply data transformations, and appreciate the need and uses of generalized linear models.
- CLO-2: Discuss about Exponential family of distributions and learn to derive likelihood based inference for such a family of distributions.
- CLO-3: Apply logistic, multinomial, Poisson and Gamma regression models.
- CLO-4: Discuss the concept of deviance, analysis of deviance, Lack-of-Fit tests in Logistic and Poisson regression, and the concept of over-dispersion.
- CLO-5: Apply the concepts of Generalized Linear Models in real life problems..
- CLO-6: Discuss and apply Quasi likelihood.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO1	3		3	3							3		
CLO2	3		3	3							3		
CLO3	3		3	3							3		
CLO4	3		3	3							3		
CLO5	3		3	3							3		
CLO6	3		3	3							3		

## Syllabus

- Unit I:** Introduction: background, review of linear models in matrix notation, model assessment, some pre-required knowledge. The exponential family of distributions: Definition and examples. Mean and variance, variance function and scale parameter. Generalized linear models (GLM): Linear predictor, link function, canonical link, Maximum likelihood estimation, iterative reweighted least squares and Fisher scoring algorithms, significance of parameter estimates, bias and consistency in GLMs, Covariance matrices of linear combinations, sampling distribution of parameter estimators, Estimating the dispersion parameter, Confidence intervals
- Unit II:** Assessing our model: Deviance, Distribution of the deviance, Deviance residuals, Examples, Partial residual/regression plots, Model goodness of fit, goodness of fit of nested models with known dispersion, goodness of fit of nested models with unknown dispersion, Akaike's Information Criterion (AIC), Collinearity and its detection, Variance inflation factors (VIFs), Testing factor variables, Interactions
- Unit III:** Normal linear regression models: least squares, analysis of variance, orthogonality of parameters, factors, interactions between factors. [2] Fitting multiple regression. Binary and Binomial data analysis: distribution and models, logistic regression models, odds ratio, one- and two-way logistic regression analysis. Multinomial regression and interpretation. Poisson count data analysis: Poisson regression models with offset, two-dimensional contingency tables, log-linear models. Gamma GLMs, alternate models, Cox-PH model.
- Unit IV:** Other GLMs- Quasi likelihoods. Extending GLMs-Random effects models, Mixed Effect models.

## References / Reading Material :

1. Dobson, A. J., An Introduction to Generalized Linear Models, Chapman & Hall 2002.
2. Krzanowski, W., An Introduction to Statistical Modelling, Edward Arnold 1998.
3. McCullagh, P. and Nelder, J. A., Generalized Linear Models, Chapman & Hall 1990.
4. Faraway, J, Extending the Linear Model with R, Chapman & Hall/CRC, 2006.
5. Agresti, A, Foundations of Linear and Generalized Linear Models, Wiley, 2015.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Introduction: background, review of linear models in matrix notation, model assessment, some pre-required knowledge. The exponential family of distributions: Definition and examples. Mean and variance, variance function and scale parameter.	1,2
Week 2	I	Generalized linear models (GLM): Linear predictor, link function, canonical link,	2,3
Week 3	I	Maximum likelihood estimation, iterative reweighted least squares and Fisher scoring algorithms, significance of parameter estimates,	2,3
Week 4	I	Bias and consistency in GLMs, Covariance matrices of linear combinations, sampling distribution of parameter estimators, Estimating the dispersion parameter, Confidence intervals	2,3
Week 5	II	Assessing our model: Deviance, Distribution of the deviance, Deviance residuals, Examples, Partial residual/regression plots,	4
Week 6	II	Model goodness of fit, goodness of fit of nested models with known dispersion, goodness of fit of nested models with unknown dispersion, Akaike's Information Criterion (AIC)	4
Week 7	II	Collinearity and its detection, Variance inflation factors (VIFs),	5
Week 8	II	Testing factor variables, Interactions	5
Week 9	III	Normal linear regression models: least squares, analysis of variance, orthogonality of parameters, factors, interactions between factors. Fitting multiple regression.	5
Week 10	III	Binary and Binomial data analysis: distribution and models, logistic regression models, odds ratio, one- and two-way logistic regression analysis. Multi-nomial regression and interpretation.	5,6
Week 11	III	Poisson count data analysis: Poisson regression models with offset, two-dimensional contingency tables, log-linear models.	5,6
Week 12	III	Gamma GLMs, alternate models, Cox-PH model.	3
Week 13	IV	Other GLMs- Quasi likelihoods.	6
Week 14	IV	Extending GLMs-Random effects models, Mixed Effect models	5,6

## 2.49 Non-Parametric Statistical Inference, IM 452/ST 586

**Course Code:** (IM 452/ST 586): I.M.Sc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Non-Parametric Statistical Inference

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** This course will provide the ability to learn the fundamentals of the most relevant nonparametric techniques for statistical inference.

**Pre-requisite Course/Knowledge (if any):** Theory of Inference – I (ST-453); Theory of Inference – II (ST-502)

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Solve inferential problems where the conditions for the traditional parametric tools to be applied are not fulfilled.
- CLO-2: Non-parametrically be able to apply one sample, two sample and multiple sample inferential problems.
- CLO-3: Determine non-parametric estimates of regression coefficients and carry out inference.
- CLO-4: Apply non-parametric density estimates.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO1	3		3	3							3		
CLO2	3		3	3							3		
CLO3	3		3	3							3		
CLO4	3		3	3							3		

## Syllabus

**Unit I:** One Sample problems. (a) Dichotomous Data: Proportion test, estimation, Bayes estimators. (b) One-sample location: Paired Replicates data. Signed Rank test, associated estimator and confidence interval. Sign test, associated estimator and confidence interval. Asymptotic procedure.

**Unit II:** Two sample problems. (a) Two-sample location Problem: Rank Sum test, associated estimator and confidence interval. Robust Rank test. (b) Two-sample dispersion: Rank test for dispersion with medians equal, Asymptotic test for dispersion – Unequal medians, (c) Test for either location or dispersion, (d) Test for general differences in two populations. (e) Two proportions: Approximate tests and confidence intervals for difference, exact Test for difference, inference for the Odds Ratio, inference for k Strata of 2X2 Tables.

Multi-sample problems. (a) One-Way Layout: Tests for general, ordered and umbrella alternatives. Test for treatments versus a control. Two and one sided tests for multiple comparisons. Contrast estimation. Simultaneous confidence intervals. (b) Two-Way Layout: Test in RCBD for general and ordered alternatives, two and one-Sided tests for multiple comparison. Test for BIBD for general alternatives, asymptotic test, test for general alternatives for data from arbitrary incomplete block design. Other tests for general alternatives, ordered alternatives, multiple comparisons, contrast estimation.

**Unit III:** Independence: Test for Independence Based on Signs, associated estimator and asymptotic confidence intervals. Test for Independence Based on Ranks, test for independence against broad alternatives.

Regression: One Regression Line (a) test for the slope (b) estimator of slope (c) Confidence interval for slope (d) estimator for intercept (e) prediction. Multiple Regression Lines (a) asymptotic test for parallelism (b) asymptotic Rank-based tests. Nonparametric Regression Analysis: Non-Rank-Based Approaches.

**Unit IV:** Density Estimation: Density Functions and Histograms, Kernel Density Estimation.

## References / Reading Material

1. Hollander, Myles; Wolfe, Douglas and Chicken, Eric., Nonparametric Statistical Methods, 3rd ed. Wiley, 2014.



2. Gibbon, Jean Dickinson and Chakraborti, Subhabrata., Nonparametric Statistical Inference, 3 ed. Marcel Dekker, 1992.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	One Sample problems-Dichotomous Data: Proportion test,	1,2
Week 2	I	One Sample problems-estimation, Bayes estimators.	1,2
Week 3	I	One-sample location: Paired Replicates data. Signed Rank test, associated estimator and confidence interval.	1,2
Week 4	I	Sign test, associated estimator and confidence interval. Asymptotic procedure.	1,2
Week 5	II	Two sample problems. (a) Two-sample location Problem: Rank Sum test, associated estimator and confidence interval. Robust Rank test. (b) Two-sample dispersion: Rank test for dispersion with medians equal, Asymptotic test for dispersion – Unequal medians, (c) Test for either location or dispersion,	1,2
Week 6	II	(d) Test for general differences in two populations. (e) Two proportions: Approximate tests and confidence intervals for difference, exact Test for difference, inference for the Odds Ratio, inference for k Strata of 2X2 Tables.	1,2
Week 7	II	Multi-sample problems. (a) One-Way Layout: Tests for general, ordered and umbrella alternatives. Test for treatments versus a control. Two and one sided tests for multiple comparisons. Contrast estimation. Simultaneous confidence intervals.	1,2
Week 8	II	b) Two-Way Layout: Test in RCBD for general and ordered alternatives, two and one-Sided tests for multiple comparison. Test for BIBD for general alternatives, asymptotic test, test for general alternatives for data from arbitrary incomplete block design. Other tests for general alternatives, ordered alternatives, multiple comparisons, contrast estimation.	1,2
Week 9	III	Independence: Test for Independence Based on Signs, associated estimator and asymptotic confidence intervals.	1,2
Week 10	III	Test for Independence Based on Ranks, test for independence against broad alternatives.	1,2
Week 11	III	Regression: One Regression Line (a) test for the slope (b) estimator of slope (c) Confidence interval for slope (d) estimator for intercept (e) prediction.	1,3
Week 12	III	Multiple Regression Lines (a) asymptotic test for parallelism (b) asymptotic Rank-based tests. Nonparametric Regression Analysis: Non-Rank-Based Approaches.	1,3
Week 13	IV	Density Estimation: Density Functions and Histograms,	1,4
Week 14	IV	Kernel Density Estimation.	1,4

## 2.50 Reliability and Survival Analysis, IM 453/ST 578

**Course Code:** (IM 453/ST 578): I.M.Sc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Reliability and Survival Analysis

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main objective is to introduce different concepts and their interpretation in reliability and survival analysis.

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

CLO-1: Discuss various statistical lifetime models.

CLO-2: Discuss various classes and their interrelations.

CLO-3: Apply non-parametric estimation in lifetime data.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	3		3								3		
CLO-2	3		3								3		
CLO-3	3		3								3		

### Syllabus

**Unit I:** Reliability concepts and measures, components and systems, coherent systems, reliability of coherent systems. Life-distributions, reliability function, hazard rate, Mean residual life, common univariate life distributions – exponential, weibull, gamma, etc..Bivariate exponential.

**Unit II:** Structure function, coherent structure Reliability and their bonds, the notion of aging; failure rate, classes of life time distributions and their relationships, Availability, maintenances through repairs and spares, multi state systems, multivariate monotone failure distributions.

**Unit III:** Estimation and testing under censoring for parametric models, testing for class properties, nonparametric models. Kaplan – Meier estimation of reliability curve, Greenwood formula, Non – parametric methods for comparison of several reliability curves, Log rank tests

**Unit IV:** Regression models in reliability, Cox PH and Accelerated failure time models; Estimation of parameters and diagnostics. conditional and partial likelihoods, asymptotic normality of estimators.

### References / Reading Material :

1. Barlow, R. E. and Proschan, F., Statistical Theory of Reliability and Life Testing, Holt, Rinehart and Winston Inc., 1975.
2. Miller, R. G., Survival Analysis, John Wiley, NY, 1981.
3. Deshpande, J. V. and Purohit, S. G., Life Time Data - Statistical Models and Methods, World Scientific, 2005.

### Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Reliability concepts and measures,	1
Week 2	I	Components and systems, coherent systems, reliability of coherent systems.	1
Week 3	I	Life-distributions, reliability function, hazard rate, Mean residual life,	1
Week 4	I	Common univariate life distributions – exponential, weibull, gamma, etc..Bivariate exponential.	1
Week 5	II	Structure function, coherent structure, Reliability and their bonds,	1, 2
Week 6	II	the notion of aging; failure rate, classes of life time distributions and their relationships,	2
Week 7	II	Availability, maintenances through repairs and spares,	2
Week 8	II	multi state systems, multivariate monotone failure distributions.	2
Week 9	III	Estimation and testing under censoring for parametric models,	2
Week 10	III	testing for class properties, nonparametric models,	1,2
Week 11	III	Kaplan – Meier estimation of reliability curve, Greenwood formula,	2, 3
Week 12	III	Non – parametric methods for comparison of several reliability curves, Log rank tests	3
Week 13	IV	Regression models in reliability, Cox PH and Accelerated failure time models; Estimation of parameters and diagnostics.	3
Week 14	IV	conditional and partial likelihoods, asymptotic normality of estimators.	3

## 2.51 Time Series, IM 454/ST 571

**Course Code:** (IM 454/ST 571): I.M.Sc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Time Series

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main purpose is to teach the theory and methods for time series modeling.

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Explain what a time series is, what stationary processes are and how this aspect helps in prediction and forecasting.
- CLO-2: Apply Correlogram and Periodogram analysis and different Smoothing methods.
- CLO-3: Discuss different stationary time series models such as MA, AR, ARMA and also non-stationary time series processes - with trend and ARIMA.
- CLO-4: Use the algorithms taught to determine the 'best' *ARMA* model that best fits a given time series data - through the *AIC* criterion and use it for prediction.
- CLO-5: Explain Volatile time series and how to model them.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO1			3	3	3						3		
CLO2	3		3	3			3				3		
CLO3	3		3	3			3				3		
CLO4	3		3	3			3				3		
CLO5	3		3	3			3				3		

## Syllabus

- Unit I:** Introduction: Examples, simple descriptive techniques, trend, seasonality, the correlogram. Common time series models, stationarity, moving average (MA), autoregressive (AR), ARMA and ARIMA models.
- Unit II:** Autocovariance, autocorrelation and partial autocorrelation functions. Estimation of parameters in AR, MA, ARMA and ARIMA models.
- Unit III:** Model fitting. AIC and BIC criteria. Residual analysis. Testing for white noise. Forecasting. Durbin-Levinson and Innovations algorithm.
- Unit IV:** Inference in frequency domain. The spectral density function, the periodogram, spectral analysis. State-space models: Dynamic linear models and the Kalman Filter.

### References / Reading Material :

1. Brockwell, Peter. J., Davis, Richard. A., Introduction to Time Series and Forecasting, Second Edition, Springer, 2002.
2. Brockwell, Peter. J., Davis, Richard. A., Time Series: Theory and Methods, Second Edition, Springer, 2006.
3. Box, G. E. P. and Jenkins, G. M., Time Series Analysis - Forecasting and control, third edition, Prentice Hall, 1994.

### Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	What are time series data? How these data are different from other data obtained. Looking at several time series data - Exchange rates, Daily Prices of crude Stock exchange closing rates, Populations of cities over many years and many more. What one can learn from such series.	1
Week 2	I	Time series as sample path of some Stochastic Process, difficulty and near impossibility of doing any 'inference' on the underlying process on whose statistical distribution nothing can not be said based on the one 'observation' - namely the series. Kolmogorov Consistency theorem.	1
Week 3	I	Tests for Series being realization of a process of iid random variables. Definitions of white Noise Process, Stationary Processes and strict Stationary process.	1
Week 4	I	Estimation and elimination of trend and Seasonality components from a time series.	1, 2
Week 5	II	Autocovariance and Autocorrelation functions of a stationary process Linear Stationary processes - from the white noise process. Proof of stationarity of such processes under absolute summability of the coefficients. The ACVFS and ACFS of such processes.	1, 2
Week 6	II	The ARMA(p; q) processes, causality and determining their ACVFs and ACFs. The PACF for a stationary processes Observing the ACF and PACF plots to understand what orders of p and q to start the model selecting process	2, 3
Week 7	II	Durbin Levinson and Innovation algorithms for prediction of later terms in a stationary time series.	2, 3
Week 8	II	Tests for stationarity of time series	1, 3
Week 9	III	Asymptotic properties of estimators of ACF, PACF and other parameters of the process	3
Week 10 & 11	III	Choosing suitable ARMA(p; q) models to fit a given time series with the aid of estimated ACVFs and ACFS and the AIC criterion.	3
Week 12	III	The ARIMA(p; d; q) process, prediction of terms in such series.	4
Week 13	IV	Selecting suitable transformations on the time series by observing the plot to attain stationarity.	4
Week 14	IV	Time series with Volatility. Suitability of the Auto regressive Conditionally Heteroskedastic ARCH() and the GARCH(; ) models for such processes. Choosing suitable such ts for a given time series with Volatility	4

## 2.52 Machine Learning Using R, IM 458/ST 573

**Course Code:** (IM 458/ST 573): I.M.Sc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** Machine Learning Using R

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main objective of this paper is to introduce some advanced statistical / machine learning techniques to extract information, visualization and knowledge using R.

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

CLO-1: Discuss different theoretical methods and practicable techniques to achieve the objectives.

CLO-2: Apply the basic concepts of machine learning techniques besides developing their ability to handle real world problems with large scale data in R.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	3										3	3	
CLO-2	3										3	3	

### Syllabus

**Unit I:** Basics of R : Data Structure, File handling, Graphics and Data Visualization, Programming. Managing and understanding data : Data description, Data processing, Dimension Reduction. Introduction to machine learning. Supervised Learning (a) Regression methods: Least-squares, Subset selection, Shrinkage,

**Unit II:** Supervised Learning- b) Classification: (LDA, Logistic, separating hyperplanes, Nearest Neighbours) c) Naive Bayes, d) Tree based classifiers d) SVM classifier, e) Neural networks (Projection pursuit regression, NN)

**Unit III:** Unsupervised Learning. a) PCA b) MDS c) ICA d) Market basket analysis using Association Rules, e) Clustering: K-means and K-medoids, hierarchical. f) Random Forests

**Unit IV:** Evaluating model performance : Measures performance for regression-  $R^2$ , F, PRESS. Evaluation for classification problem. Resampling methods : Leave-One-Out Cross- Validation, k-Fold Cross Validation. Strengths and pitfalls of cross-validation. Bootstrap methods

#### References / Reading Material :

1. Trevor Hastie, Robert Tibshirani and Jerome Friedman, The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Second Edition, Springer Series in Statistics, 2009.
2. Lantz, B., Machine learning with R. Packt Publishing, Birmingham, 2013.
3. James, Gareth, Daniela Witten, Trevor Hastie, and Robert Tibshirani, An Introduction to Statistical Learning: With Applications in R. New York: Springer, 2013.
4. Maindonald J, Braun J. Data Analysis and Graphics Using R. Cambridge University Press: Cambridge, 2003.
5. Crawley MJ, Statistics: an introduction using R. Volume 1. 1st edition. New York: John Wiley & Sons, 2005.
6. Seefeld, K. & Linder, E., Statistics Using R with Biological Examples . Department of Mathematics & Statistics, University of New Hampshire, Durham, NH, USA, 2007.
7. Vinod, H.D. (Ed.), Advances in social science research using R, Springer, New York, 2010.
8. Kleiber C, Zeileis A., Applied Econometrics with R. Springer-Verlag, New York. Forthcoming, 2006.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Basics of R : Data Structure, File handling, Graphics and Data Visualization	1
Week 2	I	Programming. Managing and understanding data : Data description,	1
Week 3	I	Data processing, Dimension Reduction	1
Week 4	I	Introduction to machine learning. Supervised Learning (a) Regression methods: Least-squares, Subset selection, Shrinkage,	1
Week 5	II	Supervised Learning- b) Classification: (LDA, Logistic, separating hyperplanes, Nearest Neighbours)	1
Week 6	II	c) Naive Bayes, d) Tree based classifiers	1, 2
Week 7	II	d) SVM classifier,	1, 2
Week 8	II	e) Neural networks (Projection pursuit regression, NN)	1, 2
Week 9	III	Unsupervised Learning. a) PCA b) MDS c) ICA	1, 2
Week 10	III	d) Market basket analysis using Association Rules,	1, 2
Week 11	III	e) Clustering: K-means and K-medoids, hierarchical.	1, 2
Week 12	III	f) Random Forests	1, 2
Week 13	IV	Evaluating model performance : Measures performance for regression- $R^2$ , F, PRESS. Evaluation for classification problem.	1, 2
Week 14	IV	Resampling methods: Leave-One-Out Cross-Validation, k-Fold Cross Validation. Strengths and pitfalls of cross-validation. Bootstrap methods	1, 2



## 2.53 Non-Linear Programming, IM 455/ST 551/MA 486/MM 590/AM 590

**Course Code:** (IM 455/ST 551/MA 486/MM 590/AM 590): I.M.Sc in Mathematical Sciences and M.Sc. in Mathematics/Applied Mathematics/ Statistics-OR

**Title of the Course:** Non-Linear Programming

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** This course introduces students to the fundamentals of nonlinear optimization theory and methods

**Pre-requisite Course/Knowledge (if any):** Basic knowledge of Nonlinear Analysis and Linear Algebra

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Discuss non-linear programming problems.
- CLO-2: Solve the non-linear programming problems.
- CLO-3: Classify the non-linear programming problems.
- CLO-4: Discuss the problems of unconstrained nonlinear programming.
- CLO-5: Discuss the necessary and sufficient conditions for the solution of unconstrained problems
- CLO-6: Apply restricted non-linear programming problems.
- CLO-7 Discuss approximate solutions of restricted problems.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO-1	3										3		
CLO-2	3										3		
CLO-3	3										3		
CLO-4	3										3		
CLO-5	3										3		
CLO-6	3										3		
CLO-7	3										3		

## Syllabus:

- Unit I:** Introduction to applications of nonlinear programming: optimal control problems, structural design, mechanical design, electrical networks, water resources management, stochastic resource allocation, location of facilities, financial engineering problems.
- Unit II:** Nonlinear programming problems, unconstrained problems, problems with inequality and equality constraints, second-order necessary and sufficient optimality conditions for constrained problems (Fritz John and Karush-Kuhn-Tucker conditions).
- Unit III:** Review of convex functions and convex optimization. Duality and optimality conditions in nonlinear programming.
- Unit IV:** Algorithms for solving NLPs: The line search methods, method of feasible directions. Focus on special application in one of the following areas: Financial engineering, supply chain management, airline optimization, production planning.

## References / Reading Material :

1. Bazaraa, M. S.; Jarvis, J. J. and Sherali, H. D., Linear Programming - and Network Flows, second edition, John Wiley, Singapore, 2003.
2. Ravindran, A. Ravi (Eds.), Operations Research and Management Science, Hand Book , CRC Press, 2009.
3. Cottle, R. W. and Lemke, C. E. (Eds), Nonlinear Programming, American Mathematical Society, Providence, RI, 1976.
4. Maindonald J, Braun J. Data Analysis and Graphics Using R. Cambridge University Press: Bertsekas, D. P., Nonlinear Programming, Athena Scientific, 1999. , 2003.

## Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Non-Linear Programming; Examples of Non-Linear programming	1
Week 2	I	Classification of Non-Linear programming problems, Convex and Concave Functions	1
Week 3	I	Unrestricted Optimization, Locally Optimal Necessary and Sufficient Conditions	1
Week 4	I	Approximate Solving Techniques on Unrestricted Optimization	1, 2
Week 5	II	Restricted Optimization; Equal case of constraints Lagrange Multipliers method	1, 2
Week 6	II	Restricted Optimization; less or equal case of constraints, Kuhn-Tucker condition	2, 3
Week 7	II	Quadratic programming Problem	2, 3
Week 8	II	Examples of Quadratic programming problem and its solutions	2, 3
Week 9	III	Convex Programming and its solving method	3, 4
Week 10	III	Examples of Convex Programming problem and its solutions	3, 4
Week 11	III	Examples of Convex Programming problem and its solutions	3, 4
Week 12	III	Separable Programming	3, 4
Week 13	IV	Approximate solving method of Restricted Optimization problems	4
Week 14	IV	Non-linear programming software	4

## 2.54 First Course on Operations Research, IM 456/ST 577

**Course Code:**( IM 456/ST 577): I.M.Sc in Mathematical Sciences and MSc Statistics-OR

**Title of the Course:** First course on Operations Research

**L-T-P per week:** 4-0-0

**Credits:** 4

**Course Overview:** The main objective of this paper is to make students acquainted with the use of optimization techniques in decision making.

**Pre-requisite Course/Knowledge (if any):** Linear Algebra, Calculus

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Convert the given real world context as a LPP.
- CLO-2: Apply the graphical, simplex tp a given LPP.
- CLO-3: Analyze the optimal solution.
- CLO-4: Evaluate the varies methods of solving LPP problems and post optimality analysis.
- CLO-5: Model Assignment and Transportation problems.
- CLO-6: Interpret the optimal solutions of LPP, AP,TP.
- CLO-7 Discuss of convexity of function.
- CLO-8 Illustrate to find an optimal solution convex function.

Mapping of CLOs with PLOs and PSOs.

	PLO										PSO		
	1	2	3	4	5	6	7	8	9	10	1	2	3
CLO1	3										3		
CLO2	3										3		
CLO3	3										3		
CLO4	3										3		
CLO5	3										3		
CLO6	3										3		
CLO7	3										3		
CLO8	3										3		

### Syllabus

- Unit I:** Introduction to Vector spaces, Matrix algebra, Formulation of LPP. Some Illustrative examples of LPP formulation. Graphical method of solving LPP in two variables
- Unit II:** Linear programming model, various forms of LPP. Foundations and theories of LPP : convex sets, feasible solution, basic solution, Basic feasible solution, optimal solution. Geometry of polyhedral
- Unit III:** Simplex method and its demonstration. Introduction of the Optimal solution. Dual of LPP, fundamental theorems of duality. Dual simplex method. Sensitivity analysis of LPP
- Unit IV:** Formation and solving Assignment Problem, Transportation Problem. Convex functions and their generalizations, Checking convexity of functions. Optimization of convex functions.

### References / Reading Material :

1. Murty, K. G., Operations Research - Deterministic Optimization Models, Prentice- Hall, 1995.
2. Murty, K. G., Linear Programming, John Wiley, NY, 1983.
3. Bazaraa, M.S.; Jarvis, J. J. and Sherali, H. D., Linear Programming and Network Flows, second edition, John Wiley, Singapore, 2003.
4. Saigal, R., Linear Programming - A Modern Integrated Analysis, Springer, 2012.
5. Bertsimas, D. and Tsitsiklis, J. N., Introduction to Linear Optimization, Athena Scientific, 1997.

6. Fourer, R.; Gay, D. M. and Kernighan, B. W., AMPL- A Modeling Language for Mathematical Programming, Brooks/Cole, 1999.

### Course Plan

Session(s)	Unit(s)	Topics	CLOs
Week 1	I	Non-Linear Programming; Examples of Non-Linear interdiction to Vector spaces, Matrix algebra,	1
Week 2	I	Formulation of LPP. Some Illustrative examples of LPP formulation.	1
Week 3	I	Graphical method of solving LPP in two variables	1
Week 4	II	Linear programming model, various forms of LPP.	1
Week 5	II	Foundations and theories of LPP : convex sets, feasible solution basic solution,	1,2
Week 6	II	Basic feasible solution, optimal solution.	1, 2
Week 7	II	Geometry of polyhedral	2, 3
Week 8	III	Simplex method and its demonstration.	2, 3
Week 9	III	Introduction of the Optimal solution	2, 3
Week 10	III	Dual of LPP, fundamental theorems of duality.	2, 3
Week 11	III	Dual simplex method. Sensitivity analysis of LPP	2, 3
Week 12	IV	formation and solving Assignment Problem	3, 4
Week 13	IV	Transportation Problem,	3, 4
Week 14	IV	Convex function and their generalizations, Checking convexity of functions. Optimization of convex functions.	4

## 2.55 Algebra, MA501/MM 801

**Name of the Academic Program:** I.M.Sc in Mathematical Sciences and Ph.D in Mathematics

**Course Code:** MA501/MM 801

**Title of the Course:** Algebra

**L-T-P per week:** 5-0-0

**Credits:** 5

**Prerequisite Course/Knowledge (if any):** Algebra- I, II, III.

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1: Able to understand the basic theory in Commutative algebra. [Knowledge, Understand(Level I, Level II)]
- CLO-2: Able to understand and solve few basic problems in Commutative Algebra. [Knowledge, Understand(Level I, Level III)]
- CLO-3: Able to apply concepts to related areas like algebraic geometry and Algebraic Number theory and Commutative Algebra. [Knowledge, Understand(Level I, Level III)]

Mapping of CLOs with PLOs and PSOs.

	PLOs						PSOs	
	1	2	3	4	5	6	1	2
CLO1	2	2		2	3	3	3	
CLO2	2	2		2	3	3	3	
CLO3	2	2		2	3	3	3	
CLO4	2	2		2	3	3	3	
CLO5	2	2		2			3	3

## Syllabus:

- **Unit I:** Review: Modules, free modules, simple and semi-simple modules, Isotypical components, Artin-Wedderburn Theorem. Free Modules, Projective modules, Injective modules, Flat Modules, finite presentation of a module, finitely presented flat module is projective.
- **Unit-II:** Artinian, Noetherian rings and modules. Jacobson and Nil radicals and the structure of Artinian rings. Modules over Noetherian rings, define support of a module and associated prime ideals of a module and showing that every associated prime ideal is in the support of a finitely generated module over Noetherian ring.
- **Unit-III:** Spectrum of a commutative ring, Krull dimension, height and coheight of a prime ideal. The relation between Krull dimension of ring  $R$  and the dimension of  $R[X]$ . Discrete Valuation rings, Fractionary ideals, Dedekind domain, Dedekind domains and their equivalent definitions.
- **Unit-IV:** Graded rings and graded modules. Hilbert-Samuel Polynomial. Ideal of definition of a Noetherian local ring. I-adic filtrations and its associated graded ring. Proving the degree of the Hilbert polynomial is independent of ideal of definition of a Noetherian local ring. Defining Chevalley dimension. Dimension theorem.

## References / Reading Material :

1. Atiya, Macdonald, Introduction to Commutative Algebra, Addison-Wesley Publishing Company, Addison-Wesley Series in Mathematics, 1969.
2. Dummit and Foote, Abstract Algebra, 3rd ed. Wiley, New York, 2003.
3. Lang, Serge, Algebra, revised third edition, Graduate Texts in Mathematics, 211, Springer-Verlag, New York, 2002.
4. Jacobson, Nathan, Basic Algebra, Volume 1, second edition, W. H. Freeman and Company, New York, 1985.
5. Jacobson, Nathan, Basic Algebra, Volume 2, second edition. W. H. Freeman and Company, New York, 1989.
6. Michael Artin, Algebra, 2nd ed. Pearson, Upper Saddle River, NJ, 2011.

7 Oscar Zariski, Pierre Samuel, Commutative Algebra I, Springer-Verlag , New York, 1975.

### Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	15	Unit-I	Understand the basics of module theory, Artin-Wedder burn theorem, Projective module, Injective module and Flat module
2	15	Unit-II	Understand the basics on the theory and structure of Artinian and Noetherian rings, Nilradical of a ring and Jacobson radical of a ring
3	15	Unit-III	Understand the spectrum of a commutative ring, the Krull dimension of a commutative ring, basic properties of Discrete Valuation rings and Dedekind domains
4	15	Unit-IV	Understand the basic properties of Graded rings, graded modules and Hilbert-Samuel polynomial.



## 2.56 Advanced Partial Differential Equations, MA 502/AM 802

**Name of the Academic Program:** I.M.Sc in Mathematical Sciences and Ph.D in Applied Mathematics

**Course Code:** MA 502/AM 802

**Title of the Course:** Advanced Partial Differential Equations

**L-T-P per week:** 5-0-0

**Credits:** 5

**Prerequisite Course/Knowledge (if any):** Real analysis -II, Measure & Integration, Functional analysis, Partial differential equations-I.

### Course Learning Outcomes (CLOs) (5 to 8)

After completion of this course successfully, the students will be able to

- CLO-1: Describe notion of distribution derivative, multiplication and convolution of distribution with test function.
- CLO-2: Explain that the Fourier transform is a isometric isomorphism on  $L^2(\mathbb{R}^n)$  and describe the notion of Fourier transform of distributions.
- CLO-3: Distinguish the behavior of the solution of the wave equation in the odd and even dimensions.
- CLO-4: Apply the maximum principle for elliptic and parabolic linear partial differential equations (PDEs).
- CLO-5: Prove the energy estimates for linear PDEs.

Mapping of CLOs with PLOs and PSOs.

	PLOs					PSOs	
	1	2	3	4	5	1	2
CLO1	2	2			3	3	
CLO2	2	2			3	3	
CLO3	2	2			3	3	
CLO4	2	2			3	3	
CLO5	2	2			3	3	

## Syllabus:

- **Unit I:** Distribution Theory: Test functions, distributions, order of the distribution, support of distributions, derivative of distributions, convolutions of distributions, fundamental solutions, the Schwartz space, Fourier Transforms, tempered distributions.
- **Unit II:** Laplace Equation in Higher dimensions: Fundamental solution, Green's functions, Poisson integral formula, mean value properties, weak and strong maximum principle, Hopf's lemma, gradient estimates, removable singularity, Perron's method.
- **Unit III:** Heat equation in Higher dimensions: Fourier transforms, fundamental solution, regularity of solutions, weak and strong maximum principle, Harnack's inequalities., radiant estimates.
- **Unit IV:** Wave equation in Higher dimensions: Solution of wave equation in higher dimension using spherical means, Huygen's principle, Duhamel's principle, energy estimates.

## References / Reading Material :

1. Kesavan S., Topics in Functional Analysis and Applications, New Age International, 2003.
2. Qing Han, A Basic Course in Partial Differential Equations, Graduate Studies in Mathematics, AMS, Vol 120, 2013.
3. Strichartz, R.S., A Guide to Distribution Theory and Fourier Transforms, World Scientific, 2003.
4. Stakgold, I., Green's Functions and Boundary Value Problems, John Wiley & Sons, 1979.
5. Hormander, L., The Analysis of Linear Partial Differential Operators, Springer, Vol 1, 1983.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	20	Unit-I	Students are able to explain weak formulation of linear differential equations and comfortably work with distributions.
2.	12	Unit-II	Students are able to analyze the well posedness of Laplace equation and properties of its solution.
3.	10	Unit-III	Students are able to analyze the well posedness of heat equation and properties of its solution.
4.	12	Unit-IV	Students are able to analyze the well posedness of wave equation and properties of its solution.

**2.57 Analysis, MA 503/IM 525/MM 803/AM 803/ST 803**

**Name of the Academic Program:** I.M.Sc in Mathematical Sciences and Ph.D in Mathematics/Applied Mathemaics/Statistics

**Course Code:** MA 503/IM 525/MM 803/AM 803/ST 803

**Title of the Course:** Analysis

**L-T-P per week:** 5-0-0

**Credits:** 5

**Prerequisite Course/Knowledge (if any):** Measure & Integration

**Course Learning Outcomes (CLOs) (5 to 8)**

After completion of this course successfully, the students will be able to

- CLO-1: Identify regular positive measures on a locally compact Hausdorff space  $X$  as a member of dual of  $C(X)$  [Level III].
- CLO-2: Apply the monotone/dominated convergence theorems and Fatou's Lemma and use them along density results to solve problems [Level III].
- CLO-3: Construct  $\sigma$ -algebra on product spaces and verify hypothesis of Fubini, Tonelli theorems [Level III].
- CLO-4: Explain the Radon-Nikodym theorem, Lebesgue's Decomposition theorem and Hahn Decomposition theorem and their applications [Level II].
- CLO-5: Distinguish between the properties of positive, real and complex measures [Level II].

Mapping of CLOs with PLOs and PSOs.

	PLOs					PSOs	
	1	2	3	4	5	1	2
CLO1	2	2				3	
CLO2	2	2				3	
CLO3	2	2				3	
CLO4	2	2				3	
CLO5	2	2				3	

## Syllabus:

- **Unit I:** Abstract Integration: Set theoretic notations and terminology, the concept of measurability, simple functions, elementary properties of measures, Arithmetic in  $[0, \infty)$ , integration of positive functions, the role played by sets of measure zero.
- **Unit II:** Positive Borel Measures: vector spaces, topological preliminaries, the Riesz representation theorem, regularity properties of Borel measures, Lebesgue measure, continuity properties of measurable functions.
- **Unit III:**  $L^p$ -spaces: Convex functions and inequalities, the  $L^p$ -spaces, approximation by continuous functions.
- **Unit IV:** Complex Measures: Total variation, absolute continuity, consequences of the Radon Nikodym theorem, bounded linear functionals on  $L^p$ , the Riesz representation theorem
- **Unit V:** Integration on product spaces and Fubini's theorem.

## References / Reading Material :

1. Halmos. P.R., Measure theory, Springer, 1950.
2. Lieb, E.H., Loss, M., Analysis, Second edition, Graduate Studies in Mathematics, AMS, 2001.
3. Rudin, W., Real and Complex Analysis, Tata McGraw-Hill Edition, 1987.

## Course Plan

S.No.	Sessions	Topics	Course Learning Outcomes
1.	15	Unit-I	Students can apply the properties of measurable sets and functions (in particular convergence theorems) to solve problems.
2.	15	Unit-II	Students can use the regularity and density results to solve problems.
3.	10	Unit-III	Students can apply the inequalities and convergence results discussed for $L^p$ spaces to estimate various quantities that they come across the study of PDEs.
4.	9	Unit-IV	The students can analyze the proofs of Lebesgue decomposition theorem and the Radon-Nikodym theorem and use them to solve problems.
5.	6	Unit-V	Students can justify the conversion of multiple integrals to the iterated integrals and they can explain the necessity of each and every assumption made in the statement of Fubini's theorem.

## 2.58 Advanced methods for statistical research (Probability and Inference), IM 521

**Course Code:** (IM 521): IMSc Statistics-OR

**Title of the Course:** Advanced methods for statistical research (Probability and Inference)

**L-T-P per week:** 5-0-0

**Credits:** 5

**Course Overview:** The main purpose is to provide the advanced theoretical foundations for probability and inference.

**Pre-requisite Course/Knowledge (if any):** PG-Probability theory, PG-Inference.

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Discuss the importance of Multivariate Central Limit Theorem, strong law of large numbers and martingales.
- CLO-2: Discuss the comparisons between bayesian and Frequentist inference.
- CLO-3: Discuss the convergence of the posterior density.
- CLO-4: Apply the concepts of the Non-parametric likelihood and Empirical likelihood.

Mapping of CLOs with PLOs and PSOs.

	PLOs										PSOs	
	1	2	3	4	5	6	7	8	9	10	1	2
CLO-1	3							3			3	
CLO-2	3							3			3	
CLO-3	3							3			3	
CLO-4	3							3			3	

## Syllabus

- Unit I:** Probability theory: Independence, Borel-Cantelli Lemma, Kolmogorov's zero-one law, strong law of large numbers, multivariate Central Limit Theorem, probability and conditional expectation, martingales.
- Unit II:** Bayesian approach: Comparisons between Bayesian and Frequentist inference, Inference, Bayesian computation, Hierarchical Bayes, Empirical Bayes Inference
- Unit III:** Convergence of the posterior density, sample techniques: types of convergence, consistency, Lindeberg-Feller theorem, asymptotic normality, rates of convergence, delta method.
- Unit IV:** Nonparametric curve estimation: Histogram and Kernel density estimators, Non-parametric likelihood, Empirical likelihood, regression.
- Unit V:** Advanced inference: locally most powerful and invariant tests, simultaneous inference, FDR.

## References / Reading Material

1. Berger, James O., Statistical decision theory and Bayesian analysis, Springer, 1985.
2. Bradley P. Carlin, Thomas A. Louis, Bayes and Empirical Bayes methods for data analysis; theory and methods. Chapman & Hall/CRC, 2000.
3. David Roxbee Cox, David Victor Hinkley, Theoretical statistics Chapman and Hall, 1982.
4. Davison, A. C. and Hinkley, D. V., Bootstrap Methods and Their Applications. Cambridge: Cambridge University Press, 1997.
5. Efron B. and Tibshirani, R. J., An Introduction to the Bootstrap. Chapman & Hall, 1993.
6. Gelman, Andrew, Bayesian data analysis. CRC Press, 2004.
7. Ghosh, J. K., Mohan Delampady, Tapas Samanta, An introduction to Bayesian analysis: theory and methods. Springer, 2006.
8. Shao Jun, Mathematical Statistics. 2nd Edition, Springer, 2003.
9. B. W. Silverman, Density estimation for statistics and data analysis. Chapman & Hall/CRC, 1998.

10. Ripley, Brian D., Stochastic simulation. WILEY-INTERSCIENCE PAPERBACK SERIES, 1987.
11. Van der Vaart, A. W., Asymptotic Statistics. Cambridge: Cambridge University Press, 1998.
12. Geoffrey J. McLachlan and 1 Thriyambakam Krishnan, The EM Algorithm and Extensions. Wiley, 1997.
13. Tanner, M. A., Tools for statistical inference. Springer, 1996.
14. Larry Wasserman , All of Statistics. Springer, 2004.
15. DasGupta, Anirban, Asymptotic Theory of Statistics and Probability, Springer, 2008.
16. David W. Scott, Multivariate Density Estimation: Theory, Practice, and Visualization, Wiley, 1992.
17. Rick Durrett, Probability: Theory and Examples, Cambridge Series in Statistical and Probabilistic Mathematics, 2010.

## 2.59 Resampling Techniques, IM 523/ST 812

**Course Code:** (IM 523/ST 812): IMSc Statistics-OR

**Title of the Course:** Resampling Techniques

**L-T-P per week:** 5-0-0

**Credits:** 5

**Course Overview:** This course requires computer work that can be done by using any of the standard softwares such as R (which is free) or Matlab (which is proprietary). The approximate ratio of time between hands on computer work and theoretical class work is 1:1.

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Discuss importance of statistical Sampling distribution, point and interval estimate
- CLO-2: Discuss importance of normal approximation and its limitations.
- CLO-3: Discuss the purpose of resampling.
- CLO-4: Apply Jackknife and Bootstrap methods and also resampling in non-i.i.d. models and linear models.

Mapping of CLOs with PLOs and PSOs.

	PLOs										PSOs	
	1	2	3	4	5	6	7	8	9	10	1	2
CLO-1	3							3			3	
CLO-2	3							3			3	
CLO-3	3							3			3	
CLO-4	3							3			3	

### Syllabus

**Unit I:** Review: Statistical model, parameters, estimates and statistics. Sampling distribution. Point and interval estimate. Estimate of mean. Pivot. t-statistic. Confidence interval. Maximum likelihood estimate (MLE). Normal distribution. Central limit theorem (CLT) and its ramifications. (3 hours)



Measures of accuracy of point and interval estimates. Normal approximation and its limitations. Shortcomings of analytic derivations, with examples. (2 hours)

Estimates of the distribution function: parametric and non-parametric MLE. (1 hour)

**Unit II:** Resampling. Purpose of resampling. Examples of, estimating the estimating variance, the sampling distribution and other features of a statistic. (1 hour)

Jackknife. Bias reduction. Estimation of variance. Delete 1 and delete d jackknives. Examples. (3 hours).

Bootstrap. Parametric and non-parametric bootstrap. Estimation of variance, estimation of distribution function. Examples. (3 hours).

**Unit III:** Comparison between bootstrap approximation and normal approximation. Examples. (2 hours).

Notions of variance consistency and distributional consistency. Jackknife distributional inconsistency. Bootstrap distributional consistency. Comparisons between bootstrap and jackknife. Examples. (4 hours).

Resampling in non-i.i.d. models: need for other resampling schemes. Examples. (2 hour)

**Unit IV:** Resampling in linear models: special emphasis on residual bootstrap and weighted bootstrap, concept of robust and efficient resampling schemes. (5 hours).

Introduction to estimating equation and generalized bootstrap. Examples. (4 hours).

### References / Reading Material :

1. Bose, Arup and Chatterjee, Snigdhanu (2018). U-Statistics,  $M_m$ -Estimators and Resampling. Volume 75 of Texts and Readings in Mathematics, Indian Edition, Hindusthan Book Agency, June 2018. [International edition Co-published by Springer, September 2018].
2. Davidson, A. C. and Hinkley D. V. (1997). Bootstrap methods and their applications.
3. Efron, B. (1982). The Jackknife, the Bootstrap and other Resampling Plans. CBMS-NSF Regional Conference Series in Applied Mathematics, No 38.

4. Efron, B. and Tibshirani, R. J. (1993). An introduction to the bootstrap. Chapman & Hall/CRC.
5. Shao, J. and T, D. (1995). The Jackknife and Bootstrap. Springer.
6. Barbe P. and Bertail P (1995). The Weighted Bootstrap. Lecture Notes in Statistics, Vol 98.
7. Gine, E. (1997). Lectures on some aspects of the bootstrap. Lecture Notes in Mathematics, Vol 1665. Springer.
8. Hall P. (1992). The Bootstrap and Edgeworth Expansion. Springer.
9. Mammen, E. (1992). When does bootstrap work? Asymptotic results and simulations. Lecture Notes in Statistics, Vol 77. Springer.

## 2.60 Research Methods in Statistics, IM 524

**Course Code:** (IM 524): IMSc Statistics-OR

**Title of the Course:** Research Methodology in Statistics

**L-T-P per week:** 5-0-0

**Credits:** 5

**Course Overview:** The main purpose is to provide the advanced resampling techniques and methods.

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

CLO-1: Discuss importance of research methodology in statistical research.

CLO-2: Apply sampling designs when the population is contiguous or non-stationary or when sampling units are not distinguishable.

CLO-3: Discuss the factorial experiments with central and axial points.

CLO-4: Apply word Processing with LaTeX and MS-Word.

Mapping of CLOs with PLOs and PSOs.

	PLOs										PSOs	
	1	2	3	4	5	6	7	8	9	10	1	2
CLO-1	3							3			3	
CLO-2	3							3			3	
CLO-3	3							3			3	
CLO-4	3							3			3	

### Syllabus

**Unit I:** Importance of research methodology in statistical research: Motivation, objectives and purpose of research. Data collection, information extraction and knowledge discovery as statistical methodology.

**Unit II:** Types of statistical research: empirical, field experiments, laboratory experiments, and secondary sources of data. Exploratory and confirmatory research. Planned and ad-hoc methods of data collection. Non-response and methods of recovering the missing response.

**Unit III:** Sampling of non-standard populations: Sampling designs when the population is contiguous or nonstationary or when sampling units are not distinguishable, are not enumerated, are affected by the process of making observation or are evasive.

**Unit IV:** Response surface methodology: Factorial experiments with central and axial points. Optimization of factor levels to maximize the response.

**Unit V:** Use of computers in statistical research: Statistical packages like SAS, SYSTAT, MINITAB and other packages like MATLAB, GAUSS, Mathematica, and Maple. R statistical computing environment.

**Unit VI:** Selection of Topic for Research, Review of Literature and its Use in Designing a Research Work-Mode of Literature Survey-Books and Monographs, Journals, Conference Proceedings, Abstracting and Indexing Journals, E- Journals/Books. Thesis Writing – Computer Application in Scientific Research, web-Searching, Scientific Articles- Statistical Data Base.

**Unit VII:** Scientific Word Processing with LaTeX and MS-Word: Article, Thesis Report and Slides Making-Power Point Features, Slide Preparation.

**Unit VIII:** History of Statistics. Statistical Heritage of India

#### **References / Reading Material :**

1. Gruijter, J., de Brus, Bierkens, M. F. P. and Knotters, M., Sampling for natural resource monitoring. Springer, 2006.
2. Thompson, S. K., Sampling, 2nd Edition. Wiley, 2002.
3. Hastie, T., Tibshirani, R. and Friedman, J., Elements of statistical learning, 2nd Edition. Springer, 2009.
4. Myers, R. H., Montgomery, D. C., and Anderson-Cook, C. M., Response surface methodology, 3rd Edition. Wiley, 2009.
5. Venables, W. N. and Ripley, B. D., Modern applied statistics with S. 4th Edition. Springer, 2002.
6. MA TLAB online manual.
7. MINIT AB online manual.
8. Mathematica online manual.
9. SAS online manual.
10. GAUSS online manual.

11. MAPLE online manual.
12. Lamport, L., LATEX: A Document Preparation System, Addison, Wesley, 2nd edition, New York, 1999.
13. Anderson, J., Durston, B.H., Poole, M., Thesis and Assignment Writing, Wiley Eastern. Ltd., New Delhi, 1970.
14. Hald, A., A History of Mathematical Statistics from 1750 to 1930, John Wiley & Sons, New York, 1998.
15. Ghosh, J.K., Mitra, S.K. and Parthasarathy, K. R., Glimpses of India's Statistical Heritage, Wiley Eastern Limited, New Delhi, 1992.

## 2.61 Linear Models and Multivariate Analysis, IM 552/ST 806

**Course Code:** (IM 552/ST 806): MSc-Statistics-OR

**Title of the Course:** Linear Models and Multivariate Analysis

**L-T-P per week:** 5-0-0

**Credits:** 5

**Course Overview:** The main purpose is to provide the advanced theoretical foundations for multivariate analysis and linear models.

**Pre-requisite Course/Knowledge (if any):** PG-Multivariate Analysis, PG-Regression Analysis.

### Course Learning Outcomes (CLO's)

After completion of this course successfully, the students will be able to

- CLO-1: Discuss the important theorems and concepts in multivariate analysis.
- CLO-2: Apply multivariate techniques appropriately, and draw appropriate conclusions.
- CLO-3: Apply advanced algebraic techniques in inference of linear models.
- CLO-4: Discuss the concept of generalized least squares.
- CLO-5: Discuss the Exponential family of distributions and apply likelihood based inference for such a family of distributions.
- CLO-6: Apply the concepts of Generalized Linear Models in real life problems.

Mapping of CLOs with PLOs and PSOs.

	PLOs										PSOs	
	1	2	3	4	5	6	7	8	9	10	1	2
CLO-1	3									3	3	
CLO-2		3									3	
CLO-3			3								3	
CLO-4	3										3	
CLO-5	3										3	
CLO-6			3									3

## Syllabus

- Unit I:** Multivariate normal vector: definition, linear transformation, characteristic function, Fisher Cochran theorem, quadratic forms
- Unit II:** Gauss Markov model with uncorrelated error, Gauss Markov theorem, estimability and identifiability, hypothesis testing of testable hypothesis, ANOVA, variance component analysis, regression, tests for model parameters.
- Unit III:** Generalized linear model: GLM with non-uncorrelated error-known as well as unknown, Logistic regression, Probit model, Cox-Proportional hazard model.

### References / Reading Material :

1. Anderson, T. W., Introduction to Multivariate Analysis, John Wiley, 1984.
2. Johnson, R.A. and Wichern. D.W., Applied multivariate Analysis. 5thAd.Prentice –Hall, 1982.
3. Sengupta, D. and Jammalamadaka Rao, S., Linear Models-An Integrated Approach, World Scientific, 2003.
4. Rao, C.R., Linear Statistical Inferences and its Applications, 2nd ed., John Wiley & Sons, 1973.
5. McCullagh, P. and Nelder, J. A., Generalized Linear Models, Chapman & Hall 1990.
6. Dobson, A. J., An Introduction to Generalized Linear Models, Chapman & Hall 2002.

## 2.62 Research Methodology, MM 808/AM 808/ST 808

**Name of the Academic Program:** Ph.D in Mathematics/Applied Mathematics/Statistics

**Course Code:** MM 808/AM 808/ST 808

**Title of the Course:** Research Methodology

**L-T-P per week:** 4-0-0

**Credits:** 4

**Prerequisite Course/Knowledge (if any):** NIL

**Syllabus:**

- Objectives of research, Types of research, Research approaches, Importance of knowing how research is done, Research process.
- Literature survey, Finding a research problem, How to read research article (a case study), Methods and processes for solving the problem.
- Structure and components of research reports, Technical report writing and presentation in LATEX(a case study).
- Where to publish, How to find a good journal, Journal indexing, Information about various mathematical and statistical societies, Information about seminars, conferences and workshops.

**References / Reading Material :**

1. Kothari, C. R. (2004). Research Methodology: Methods and Techniques . New Age International,
2. Krantz, S. G. (2017). A Primer of Mathematical Writing: Second Edition. American Mathematical Society.
3. Higham, N. J. (1998). Handbook of Writing for the Mathematical Sciences. Society for Industrial and Applied Mathematics.
4. Oetiker, T., Partl, H., Hyna, I., and Schlegl, E. (1995). The not so short introduction to LATEX2". <http://www.tex.ac.uk/tex-archive/info/lshort>.



## 2.63 Research and Publications Ethics, MM 831/AM 831/ST 831

**Name of the Academic Program:** Ph.D in Mathematics/Applied Mathematics/Statistics

**Course Code:** MM 811/AM 811/ST 811

**Title of the Course:** Research and Publication ethics

**L-T-P per week:** 2-0-0

**Credits:** 2

**Prerequisite Course/Knowledge (if any):** NIL

**Syllabus:**

- **Philosophy and Ethics:**
  - (a) Introduction to philosophy: definition, nature and scope, concept, branches.
  - (b) Ethics: definitions, moral philosophy, nature of moral judgments and reactions.
- **Scientific Conduct:**
  - (a) Ethics with respect to science and research.
  - (b) Intellectual honesty and research integrity.
  - (c) Scientific misconducts: falsification, fabrication and plagiarism.
  - (d) Redundant publications: duplicate and overlapping publications, salami slicing.
  - (e) Selective reporting and misrepresentation of data.
- **Publication Ethics:**
  - (a) Publication ethics: definition, introduction and importance.
  - (b) Best practices/standards setting initiatives and guidelines: COPE, WAME, etc.
  - (c) Conflicts of interest.
  - (d) Publication misconduct: definition, concept, problems that lead to unethical behavior and vice versa, types.
  - (e) Violation of publication ethics, authorship and contributorship.

- (f) Identification of publication misconduct, complaints and appeals.
- (g) Predatory publishers and journals.
- **Open Access Publishing:**
  - (a) Open access publications and initiatives.
  - (b) SHERPA/RoMEO online resource to check publisher copyright and self-archiving policies.
  - (c) Software tool to identify predatory publications developed by SPPU.
  - (d) Journal finder/journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc.
- **Publication Misconduct:**
  - (a) Group Discussion
    - i. Subject specific ethical issues, FEP, author.
    - ii. Conflicts of interest.
    - iii. Complaints and appeals: examples and fraud from India and abroad.
  - (b) Software tools: Use of plagiarism software like Turnitin, Urkund and other open source software tools.
- **Databases and Research Metrics:**
  - (a) Databases:
    - i. Indexing databases.
    - ii. Citation databases: Web of Science, Scopus, etc.
  - (b) Research Metrics
    - i. Impact factor of journal as per Journal Citation Report, SNIP, SJR, IPP, Cite Score.
    - ii. Metrics: h-index, g-index, i10 index, allmetrics

### **References / Reading Material :**

1. Brid, A., Philosophy of Science, Routledge, 2006.
2. MacIntyre, Alasdair, A short history of ethics, London, 1967.
3. Chaddah, P., Ethics in competitive research: Do not getscooped; do not get plagiarized, 2018, ISBN-978-9387480865.
4. Resnik, D. B., What is ethics in research and why is it important, 2011, National Institute of Environmental Health Sciences, 1-10. Retrieved from <https://www.niehs.nih.gov/research/resources/bioethics/whatis/index.cfm>

5. Beall, J., Predatory publishers are corrupting open access, *Nature*, 489 (7415) (2012) 179-179.
6. National Academy of Sciences, National Academy of Engineering and Institute of Medicine, *On being a Scientist: A guide to responsible conduct in research*, 2009, Third edition, National Academic Press.
7. INSA (Indian National Science Academy), *Ethics in science education, research and governance*, 2019, ISBN:978-81-939482-1-7. <http://www.insaindia.res.in/pdf/Ethics-Book.pdf>